Identifying the Uncertain Model Parameter of a Steam Turbine System

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ABSTRACT

The transformation method (TM) of fuzzy arithmetic is aimed at simulation and analysis of a system. The aim of this paper is to use fuzzy arithmetic based on the TM on a state space of a steam turbine system. The model is then used to identify the degree of influence of each parameter on the system. Simulation and analysis of the system are presented in this paper.

Keywords: Fuzzy arithmetic, uncertain model parameter, steam turbine system

INTRODUCTION

A power generation plant consists of a of a steam turbine, generator, condenser, boiler and feed water system (Wagner & Priluck, 1982; Mukherjee, 1984; De Mello, 1991a). A combined cycle power plant is a large-scale electric power generation plant in which electricity is obtained from both gas and steam turbines. Energy is transferred in the form of heat or gaseous flow in each section of the turbine (Ordys et al., 2012). A combination of gas and a steam turbine gives the best output for a high efficiency thermal process (Kehlhofer et al., 2009).

There are several ways to increase output of a steam turbine, namely by increasing the amount of energy that goes into the steam turbine or improving its effectiveness. The former method is easier using a large turbine with huge wheels and vanes. However, the drawback is the efficiency of the steam turbine itself. Therefore, identifying inputs which have the highest degree of influences is paramount in order to maximise energy production.

The general transformation method (TM) of fuzzy arithmetic has proven to be a realistic approach in determining these parameters.
is a selection method to reduce the number of parameters of a given system. In this way, the influence of the uncertain parameters can be determined, thus simplifying and enhancing the system simulations and analysis.

The model inputs are represented by fuzzy numbers $\bar{p}$, $\sigma_1$ and $\sigma_r$, which denote the worst deviations from the mean value in quasi-Gaussian membership function distribution form.

$$\bar{p} = g fn^*(\bar{x}, \sigma_1, \sigma_r)$$  \[1\]

The quasi-Gaussian fuzzy number $\bar{p} \in \mathbb{P}'(R)$ with the membership function is expressed (Hanss, 2005) by:

$$\mu_{\bar{p}}(x) = \begin{cases} 
0 & \text{for } x \leq \bar{x} - 3\sigma_1 \\
\exp\left[-\frac{(x-\bar{x})^2}{2\sigma_1^2}\right] & \text{for } \bar{x} - 3\sigma_1 < x < \bar{x} \\
\exp\left[-\frac{(x-\bar{x})^2}{2\sigma_r^2}\right] & \text{for } \bar{x} < x < \bar{x} + 3\sigma_r \\
0 & \text{for } x \geq \bar{x} + 3\sigma_r 
\end{cases} \quad \forall x \in \mathbb{R} \[2\]

The quasi-Gaussian fuzzy number consists of Gaussian fuzzy number that is truncated as and $x < \bar{x} - 3\sigma_1$ and $x > \bar{x} + 3\sigma_r$ respectively.

The state space modelling of steam turbine is presented in Section 2, followed by the transformation method of fuzzy arithmetic in Section 3, and its implementation in Section 4.

**STATE SPACE MODELLING OF A STEAM TURBINE**

The mathematical modelling of a steam turbine was first presented by Ray (1980). The model considered impulses and reaction stages of the steam turbine. De mello(1991b) and Report (1973) meanwhile presented simple linear models of the steam turbine and the importance of non-linearity. Below are the assumptions of these models:

- Superheated steam is considered as an ideal gas.
- The steam turbine is divided into three sections: high pressure (HP), intermediate pressure (IP), and low pressure (LP).
- Kinetic energy at inlets of each stage is negligible.
- The energy stored at each stage is lumped.

The complete turbine stage (HP, IP or LP) comprises the number of impulse and reaction stages in series. The impulse or reactions at each stage in this connection are expressed in the following equation (Khan et al., 2012).

$$\frac{d}{dt}(X_o) = \frac{1}{v}(w_{in}h_{in} - w_{ou}h_o)$$  \[3\]

$$\frac{d}{dt}(w_{ou}) = \frac{1}{\tau_s}(w_{in} - w_{ou})$$  \[4\]
\[
\frac{d}{dt}(\rho_{ou}) = \frac{1}{v}(w_{in} - w_{ou}) \tag{5}
\]

The ideal gas and nozzle are presented by Equations [6-9]:

\[
T_0 = \frac{h_o - h_{in}}{c_p} + T_{in} \quad \text{and} \quad p_o = R\rho_o T_0 \tag{6}
\]

\[
\left( r^{\frac{2}{m}} - r^{\frac{m+1}{m}} \right) = \frac{w_{ou}^2}{A^2\rho_o p_o} \left( \frac{m-1}{2\eta \infty m} \right) \tag{7}
\]

where \( m = \frac{\gamma}{\gamma - \eta \infty (1 - \gamma)} \)

\[
\frac{T_{ou}}{T_0} = \left[ \frac{p_{ou}}{p_o} \right]^{\eta \infty (\frac{\gamma - 1}{\gamma})} \quad \text{where} \quad \gamma = \frac{c_p}{c_p - R} = \frac{c_p}{c_v} \tag{8}
\]

\[
\Delta h_1 = c_p T_0 \left( r^{c_p} - 1 \right) \tag{9}
\]

The state space model is formulated as below:

\[
\frac{d}{dt}(X_o) = \frac{w_{in} c_p}{v} T_{in} + \frac{w_{in} h_o}{v} - \frac{w_{in}}{v} \cdot \frac{c_p}{R \rho_o} \cdot \frac{p_{in} T_{in}}{T_{ou}} - \frac{w_{ou} h_o}{v} \tag{10}
\]

\[
\frac{d}{dt}(w_{ou}) = \frac{1}{\tau_s} \left[ W_{in} - 2\eta \infty \rho_0 \left( r^{\frac{2}{m}} - r^{\frac{m+1}{m}} \right) \frac{A X_o}{h_o} \right] \tag{11}
\]

\[
\frac{d}{dt}(\rho_{ou}) = \frac{1}{v} \left[ W_{in} - \frac{A \rho_{ou}}{r} 2\eta \infty \rho_0 \left( r^{\frac{2}{m}} - r^{\frac{m+1}{m}} \right) \frac{A X_o}{h_o} \right] \tag{12}
\]

\[
\rho_{ou} = \frac{r R T_o X_o}{h_o} \tag{13}
\]

\[
T_{ou} = \frac{(c_p T_{ou} - h_{ou}) \rho_{ou}^{\frac{\eta \infty (\gamma - 1)}{\gamma}}}{r c_p p_o} + \frac{X_o r^{\frac{\eta \infty (\gamma - 1)}{\gamma}}}{c_p \rho_o} \tag{14}
\]
From equation [3-15] the state space equation is expressed as below:

\[
\begin{bmatrix}
X_o' \\
W_{ou}' \\
\rho_{ou}'
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{-h_o}{v} & 0 \\
\frac{\frac{2\eta_{oo}m_{p_0}}{v} \left( \frac{Z}{m} - \frac{m+1}{m} \right) \frac{(m-1)p_0}{h_0 \tau_s}}{A} & 0 & 0 \\
0 & 0 & \frac{2\eta_{oo}m_{p_0}}{v} \frac{\left( \frac{Z}{m} - \frac{m+1}{m} \right) \frac{(m-1)p_0}{rV}}{A}
\end{bmatrix}
\begin{bmatrix}
X_o \\
W_{ou} \\
\rho_{ou}
\end{bmatrix} +
\begin{bmatrix}
\frac{h_o}{v} \\
\frac{w_{in}}{v} \\
\frac{1}{\tau_s} \\
\frac{1}{v}
\end{bmatrix}
\begin{bmatrix}
w_{in} \\
T_{in} \\
P_{in} \\
T_{in}
\end{bmatrix}
\]

Equations [16 and 17] form the state space model of a section of the steam turbine. The model can be applied to all sections of the steam turbine.

**TRANSFORMATION METHOD OF FUZZY ARITHMETIC**

The advanced fuzzy arithmetic can be considered as an elimination of restrictions in its area of applications by Hanss (2005). Therefore, it can be used to evaluate fuzzy rational expressions and can also be applied to simulate static or dynamic systems with fuzzy valued parameters.
Transformation method is basically available in two forms: general and in reduced form. Both methods are practical instruments for simulation and analysis of any system with uncertain model parameters (Hanss, 2002).

The problem with uncertainties can be solved with the implementation of transformation method of fuzzy arithmetic, since the fuzzy valued result of the problem only shows the overall influence of all the uncertain parameters. However, the degree of influence of the different uncertain model parameters are certainly not equal in general. In this regard, the percentages to which the \( n \) uncertain parameters of the system contribute to the overall uncertainty of the system can be determined. The coefficients for general transformation method are proposed by Hanss and Nehls (2000, 2001), and Hanss (2002, 2005).

### Simulation of a system with uncertain parameters: General transformation method

The uncertain parameters can be represented by fuzzy numbers \( \tilde{p}_i \) with \( i = 1, 2, \ldots, n \).

\[
P_i = \{X_i^{(0)}, X_i^{(1)}, \ldots, X_i^{(m)}\}
\]  

\[
X_i^{(j)} = [a_i^{(j)}, b_i^{(j)}], \quad a_i^{(j)} \leq b_i^{(j)}, 
\]

\( i = 1, 2, \ldots, n, \quad j = 0, 1, \ldots, m. \)

A fuzzy parameterised model is expected to show non-monotonic behaviour with respect to \( n \), for fuzzy value parameter \( \tilde{p}_i \) with \( i = 1, 2, \ldots, n \). The intervals \( X_i^{(j)} \), \( i = 1, 2, \ldots, n, \quad j = 0, 1, \ldots, m-2 \), are considered for the transformation scheme. The intervals are now transformed into arrays \( \hat{X}_i^{(j)} \) as below:

\[
X_i^{(j)} = \left( (Y_{1,i}^{(j)}, Y_{2,i}^{(j)}, \ldots, Y_{(m+1-j),i}^{(j)}), \ldots, (Y_{(m-1),i}^{(j)}, Y_{2,i}^{(j)}, \ldots, Y_{m+1-j,i}^{(j)}) \right)
\]  

\[
Y_{l,i}^{(j)} = \left( c_{l,i}^{(j)}, \ldots, c_{l,i}^{(j)} \right), \quad (m+1-j)^{n-i} \text{ elements}
\]

\[
c_{l,i}^{(j)} = \begin{cases} 
    a_i^{(j)} & \text{for } l = 1 \text{ and } j = 0, 1, \ldots, m, \\
    \frac{1}{2}(c_{l-1,i}^{(j+1)} + c_{l,i}^{(j+1)}) & \text{for } l = 2, 3, \ldots, m - j \text{ and } j = 0, 1, \ldots, m - 2, \\
    b_i^{(j)} & \text{for } l = m - j + 1 \text{ and } j = 0, 1, \ldots, m,
\end{cases}
\]

The arithmetical expression of \( F \) is expressed as:

\[
\tilde{q} = F(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_n)
\]
The expression is evaluated separately at each \(2^n\) positions of the combination using conventional arithmetic for crisp numbers. The result of the problem can be expressed in its decomposed and transformed form by the combination \(\hat{Z}(j), j = 0, 1, \ldots, m\), the \(k\)th element \(k\hat{Z}(j)\) of the array \(\hat{Z}(j)\) as below:

\[
k\hat{Z}(j) = F\left(kx_1(j), kx_2(j), \ldots, kx_n(j)\right)
k = 1, 2, \ldots, 2^n.
\]

Finally, the fuzzy-valued result \(\hat{q}\) of the expression can be achieved in its decomposed form

\[
Z(j) = [a(j), b(j)], j = 0, 1, \ldots, m
\]

By retransforming \(\hat{Z}(j)\) with a correction procedure namely by the recursive formulae

\[
a(j) = \min_k (a(j+1), k\hat{Z}(j)),
\]

\[
b(j) = \max_k (b(j+1), k\hat{Z}(j)), j = 0, 1, \ldots, m-1,
\]

\[
a(m) = \min_k (k\hat{Z}(m)) = \max_k (k\hat{Z}(m)) = b(m)
\]

Analysis of system with uncertain parameters: General transformation method

The coefficients \(\eta_i(j), i = 1, 2, \ldots, n, j = 0, 1, \ldots, m-1\), are determined using the following equation:

\[
\eta_i(j) = \frac{1}{(m-j+1)^{n-1}(b_i(j)-a_i(j))} \sum_{k=1}^{(m-j+1)^{n-1}} \sum_{l=1}^{(m-j+1)^{l-1}} \left(s_2\hat{Z}(j) - s_1\hat{Z}(j)\right)
\]

\[
s_1(k, l) = k + (l - 1)(m - j + 1)^{n-l+1}
\]

\[
s_2(k, l) = k + [(m - j + 1)l - 1](m - j + 1)^{n-l}
\]

The values \(a_i(j)\) and \(b_i(j)\) denote the lower and upper bound of the interval \(X_i(j)\), and \(k\hat{Z}(j)\) is the \(k\)th element of the array \(\hat{Z}(j)\). The coefficients \(\eta_i(j)\) can be interpreted as gain factors which express the effect of the uncertainty of the \(i\)th parameter on the uncertainty of the output \(z\) of the problem at the membership level \(\mu_j\). The mean gain factor \(\bar{\eta}_i(j)\) is computed such that:

\[
\bar{\eta}_i = \frac{\sum_{j=1}^{m-1} \mu_j \eta_i(j)}{\sum_{j=1}^{m-1} \mu_j}
\]

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Finally, the degree of influence, namely the normalised values $\rho_i$, is determined for $i = 1, 2, \ldots, n$, using

$$\rho_i = \frac{\sum_{j=1}^{m-1} \mu_j \eta_i^{(j)} (a_i^{(j)} + b_i^{(j)})}{\sum_{q=1}^{n-1} \sum_{j=1}^{m-1} \mu_j \eta_q^{(j)} (a_q^{(j)} + b_q^{(j)})}$$  \[33\]

$$\sum_{i=1}^{n} \rho_i = 1$$  \[34\]

Thus, the values $\rho_i$ give the influence of the $i$th varying parameter $\tilde{p}_i$ on the overall variation $\tilde{q}$ of the system output $z$, assuming that every parameter is to be varied relatively to the same extent.

From the geometrical point of view, it can be described as multiple evaluations of the system for different set of crisp parameter with the coordinates of points on the $(n-1)$-dimensional hyper surfaces of a number of $m+1$ $n$-dimensional cuboids. Only $2^n$ vertex points of the $n$-dimensional cuboids are considered for the evaluation of the problem and additional point on the hyper surfaces are taken into account for general transformation method (Hanss, 2002).

APPLICATION TO A STEAM TURBINE SYSTEM

Simulation for the parameter of HP, IP and LP of the steam turbine are considered as independent parameters. The uncertain model parameters and the model inputs are represented by quasi-Gaussian distribution fuzzy numbers (Table 1).

Table 1
Fuzzified input parameters for HP, IP and LP of the steam turbine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
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</tr>
<tr>
<td>$\tilde{p}<em>1 = w</em>{in}$</td>
<td>a</td>
<td>10</td>
<td>10.9</td>
<td>11.2</td>
<td>11.4</td>
<td>11.6</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>14</td>
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<td>12.8</td>
<td>12.6</td>
<td>12.4</td>
</tr>
<tr>
<td>$\tilde{p}<em>2 = p</em>{in}$</td>
<td>a</td>
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<td>4520000</td>
<td>4521000</td>
<td>4522000</td>
<td>4523000</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>4535000</td>
<td>4530000</td>
<td>4529000</td>
<td>4528000</td>
<td>4527000</td>
</tr>
<tr>
<td>$\tilde{p}<em>3 = T</em>{in}$</td>
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<td>716.6</td>
<td>716.9</td>
<td>717.1</td>
<td>717.3</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>720</td>
<td>718.8</td>
<td>718.5</td>
<td>718.3</td>
<td>718.1</td>
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<tr>
<td>$\tilde{p}<em>4 = h</em>{in}$</td>
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<td>3308000</td>
<td>3309000</td>
<td>3310000</td>
</tr>
<tr>
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<td>b</td>
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<td>3317000</td>
<td>3316000</td>
<td>3315000</td>
<td>3314000</td>
</tr>
<tr>
<td>$\tilde{p}<em>5 = \rho</em>{in}$</td>
<td>a</td>
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<td>12.4</td>
<td>12.7</td>
<td>13</td>
<td>13.2</td>
</tr>
<tr>
<td></td>
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<td>14.9</td>
<td>14.6</td>
<td>14.4</td>
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Table 1 (continue)

<table>
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<th>Parameter</th>
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<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
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<td>1276000</td>
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<td>1340000</td>
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<td>726.4</td>
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<td>3331000</td>
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<tr>
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<td>3067000</td>
<td>3068000</td>
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</table>

Figure 1. (a) Triangular Fuzzy Number; (b) Gaussian Fuzzy Number
The uncertain steam turbine of HP, IP and LP can be simulated using transformation method in general form and considering $\rho_{ou}$, $T_{ou}$ as the uncertain output $\bar{q}(t)$. The results of the simulation of the model with a decomposition number $m = 5$, can be obtained as a fuzzy-valued output of the model. The graph is plotted against time and simulated by MATLAB. The contour plot of the uncertain steam turbine of HP, IP and LP model with the membership $\mu$ as a contour parameter in steps of $\Delta \mu = 0.2$ with (a) $\rho_{ou}$ (b) $T_{ou}$ and (c) $P$ is presented by Figures 2-4.

Figure 2. Contour Plot of The Uncertain Steam Turbine of HP Model with $\mu$ as a Contour Parameter in Steps of $\Delta \mu = 0.2$ with (a) $\rho_{ou}$; (b) $T_{ou}$; and (c) $P$
Figure 3. Contour Plot of the Uncertain Steam Turbine of IP Model with $\mu$ as a Contour Parameter in Steps of $\Delta\mu = 0.2$ with (a) $P_{OU}$, (b) $T_{OU}$, and (c) $P$. 
Figure 4. Contour Plot of the Uncertain Steam Turbine of LP Model with $\mu$ as a Contour Parameter in steps of $\Delta \mu = 0.2$ with (a) $P_{ou}$, (b) $T_{ou}$, and (c) $P$.
On closer examination (Figure 2), the worst case of interval for HP section of steam turbine at \( t = 138.5 \) seconds is (a) \( \rho_{ou} = 4.15 \times 10^{27} \text{ Pa} \) (b) \( T_{ou} = 8.07 \times 10^{24} \, ^\circ\text{K} \) and (c) \( P = 2.245 \times 10^{28} \, W \). The initial condition for HP section of steam turbine is \( t = 110 \) seconds.

Based on Figure 3, the worst case of interval for IP section of steam turbine at \( t = 69.35 \) seconds is (a) \( \rho_{ou} = 3.151 \times 10^{36} \text{ Pa} \) (b) \( T_{ou} = 2.043 \times 10^{22} \, ^\circ\text{K} \) and (c) \( P = 2.822 \times 10^{25} \, W \). The initial condition for IP section of steam turbine is \( t = 50 \) seconds.

The worst case of interval for LP section (see Figure 4) of steam turbine at \( t = 79.67 \) seconds is (a) \( \rho_{ou} = 6.53 \times 25 \text{ Pa} \) (b) \( T_{ou} = 7.81 \times 10^{24} \, ^\circ\text{K} \) and (c) \( P = 1.314 \times 10^{28} \, W \). The initial condition for LP section of steam turbine is \( t = 60 \) seconds.

As a result of analysing the overall steam turbine model, the parameters \( \rho_1, \rho_2, \ldots, \rho_5 \) to be identified in the final state space model with \( n = 5 \) are independent uncertain parameters.

The settings for the mean value \( (\bar{x}_i) \) and the standard deviation \( (\sigma_i) \) of the fuzzy parameters are listed in Table 2. The high values for the standard deviations are assumed to be a large uncertainty of the model parameters (Hanss & Nehls, 2001). The following is the settings for the mean values and the standard deviations of the uncertain model parameters.

**Table 2**

*Settings for the mean values \((\bar{x}_i)\) and the standard deviations \((\sigma_i)\) of the uncertain model parameters*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>( \bar{x} )</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High Pressure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_1 )</td>
<td>kg/s</td>
<td>12</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>Pa</td>
<td>4525016.67</td>
<td>0.000738</td>
<td>0.00074</td>
</tr>
<tr>
<td>( \rho_3 )</td>
<td>( ^\circ\text{K} )</td>
<td>717.67</td>
<td>0.0013</td>
<td>0.0011</td>
</tr>
<tr>
<td>( \rho_4 )</td>
<td>J/kg</td>
<td>3311816.67</td>
<td>0.0019</td>
<td>0.001</td>
</tr>
<tr>
<td>( \rho_5 )</td>
<td>kg/m(^3)</td>
<td>13.64</td>
<td>0.065</td>
<td>0.06</td>
</tr>
<tr>
<td><strong>Intermediate Pressure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_1 )</td>
<td>kg/s</td>
<td>10.47</td>
<td>0.047</td>
<td>0.049</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>Pa</td>
<td>1302816.67</td>
<td>0.0264</td>
<td>0.0249</td>
</tr>
<tr>
<td>( \rho_3 )</td>
<td>( ^\circ\text{K} )</td>
<td>726.88</td>
<td>0.0032</td>
<td>0.0015</td>
</tr>
<tr>
<td>( \rho_4 )</td>
<td>J/kg</td>
<td>3332816.67</td>
<td>0.0013</td>
<td>0.00072</td>
</tr>
<tr>
<td>( \rho_5 )</td>
<td>kg/m(^3)</td>
<td>4.07</td>
<td>0.33</td>
<td>0.49</td>
</tr>
<tr>
<td><strong>Low Pressure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_1 )</td>
<td>kg/s</td>
<td>10.45</td>
<td>0.047</td>
<td>0.05</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>Pa</td>
<td>477275</td>
<td>0.054</td>
<td>0.051</td>
</tr>
<tr>
<td>( \rho_3 )</td>
<td>( ^\circ\text{K} )</td>
<td>608.74</td>
<td>0.011</td>
<td>0.0062</td>
</tr>
<tr>
<td>( \rho_4 )</td>
<td>J/kg</td>
<td>307115</td>
<td>0.00122</td>
<td>0.00151</td>
</tr>
<tr>
<td>( \rho_5 )</td>
<td>kg/m(^3)</td>
<td>1.58</td>
<td>0.33</td>
<td>0.3</td>
</tr>
</tbody>
</table>
The non-monotonic properties are likely to appear as a consequence of linear elements in the model equations [3-17]. The uncertainty of the steam turbine model is then analysed using an appropriate form of general transformation method. The analysis can be evaluated and the degree of influence can be obtained.

Finally, the result of the analysis model shows the relative influence $\rho_i$ of the uncertain parameters $\rho_i$, $i = 1, 2, ..., 5$ on the uncertainty of $\bar{q}(t) = T_{out}(t)$ (see Table 3) for every section in the steam turbine model.

Table 3

<table>
<thead>
<tr>
<th>$i$</th>
<th>High Pressure</th>
<th>Intermediate Pressure</th>
<th>Low Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.16</td>
<td>5.1335</td>
<td>12.4126</td>
</tr>
<tr>
<td>2</td>
<td>47.00</td>
<td>3.7212</td>
<td>4.3130</td>
</tr>
<tr>
<td>3</td>
<td>36.05</td>
<td>39.3510</td>
<td>0.0250</td>
</tr>
<tr>
<td>4</td>
<td>15.58</td>
<td>51.7488</td>
<td>82.8612</td>
</tr>
<tr>
<td>5</td>
<td>0.21</td>
<td>0.0455</td>
<td>0.3883</td>
</tr>
</tbody>
</table>

The evaluation of the degree of influence $\rho_i$, $i = 1, 2, ..., 5$, for all sections of the steam turbine indicates that each of the parameters are considered to be varied to the same extent relative to its peak value.

For the HP section, the overall uncertainty of the model output is induced by the uncertainty of $\rho_2$, $\rho_3$, and $\rho_4$. It shows a moderate impact. The rest of the uncertainties $\rho_1$ and $\rho_5$ have a low impact and can be considered negligible.

Next, for the IP section, about half of the overall uncertainty of the model output are induced by the uncertainty of $\rho_4$ and followed by the uncertainties $\rho_3$ of which show moderate impact. The rest of the uncertainties of $\rho_1$, $\rho_2$ and $\rho_5$ only show low impact and can be considered negligible.

Moreover, for the LP section the uncertain parameter of $\rho_4$ has a high degree of influence on an overall uncertainty of the output model. However, the uncertain parameters $\rho_1$ and $\rho_2$ only have a low impact and $\rho_3$ and $\rho_5$ are negligible.

CONCLUSIONS

The transformation method was applied to the steam turbine system. It showed significant reliability with minimal computing time.

ACKNOWLEDGEMENTS

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REFERENCES


### APPENDIX

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Definition</th>
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</thead>
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<tr>
<td></td>
<td></td>
<td><strong>High Pressure</strong></td>
</tr>
<tr>
<td>( W_{in} )</td>
<td>( \text{kg/s} )</td>
<td>Inlet steam flow from boiler superheat section</td>
</tr>
<tr>
<td>( P_{in} )</td>
<td>( \text{Pa} )</td>
<td>Inlet steam pressure</td>
</tr>
<tr>
<td>( T_{in} )</td>
<td>( ^{\circ} \text{K} )</td>
<td>Inlet steam flow temperature</td>
</tr>
<tr>
<td>( h_{in} )</td>
<td>( \text{J/kg} )</td>
<td>Inlet steam flow enthalpy</td>
</tr>
<tr>
<td>( \rho_{in} )</td>
<td>( \text{Kg/m}^3 )</td>
<td>Inlet steam flow density</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Intermediate Pressure</strong></td>
</tr>
<tr>
<td>( W_{oIP} )</td>
<td>( \text{kg/s} )</td>
<td>Inlet steam flow from boiler reheater section</td>
</tr>
<tr>
<td>( P_{o} )</td>
<td>( \text{Pa} )</td>
<td>Inlet steam pressure</td>
</tr>
<tr>
<td>( T_{o} )</td>
<td>( ^{\circ} \text{K} )</td>
<td>Inlet steam flow temperature</td>
</tr>
<tr>
<td>( h_{o} )</td>
<td>( \text{J/kg} )</td>
<td>Inlet steam flow enthalpy</td>
</tr>
<tr>
<td>( \rho_{o} )</td>
<td>( \text{Kg/m}^3 )</td>
<td>Inlet steam flow density</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Low Pressure</strong></td>
</tr>
<tr>
<td>( W_{oLP} )</td>
<td>( \text{kg/s} )</td>
<td>Inlet steam flow from IP section</td>
</tr>
<tr>
<td>( P_{oLP} )</td>
<td>( \text{Pa} )</td>
<td>Inlet steam pressure</td>
</tr>
<tr>
<td>( T_{oLP} )</td>
<td>( ^{\circ} \text{K} )</td>
<td>Inlet steam flow temperature</td>
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<tr>
<td>( h_{oLP} )</td>
<td>( \text{J/kg} )</td>
<td>Inlet steam flow enthalpy</td>
</tr>
<tr>
<td>( \rho_{oLP} )</td>
<td>( \text{Kg/m}^3 )</td>
<td>Inlet steam flow density</td>
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