Theoretical Development of Biaxial Fabric Prestressed Composites under Tension-Tension Fatigue Loading

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ABSTRACT

The improvement of the composite material against fatigue loading is of a great interest. In this study, the classical lamination theory of laminated composite was developed in order to include the effect of fibre prestressing on the composite’s fatigue life when it was subjected to tension-tension fatigue loading. The biaxial fabric prestress term of the plain-weave composite (E-glass/polyester) was included in the theory and simplified. The overall tensile stress within the composite lamina was reduced by inducing compressive residual stress imparted from releasing the fibre pretension load. The fatigue life of the prestressed E-glass/polyester composite lamina was prolonged 36 times compared to the non-prestressed counterparts when the fabric was biaxially prestressed with 100 MPa.

Keywords: Biaxial fabric prestress, development of classical lamination theory, plain-weave fabric, residual stresses, tension-tension fatigue

INTRODUCTION

There is a need for high strength and lightweight materials in applications, such as in the aerospace, civil, automotive and sporting goods industries. Therefore, fibre-reinforced composite materials have sparked interest due to their high strength-to-weight ratio in comparison with most metals. The cost of composites, for instance, the cost of fibreglass-reinforced polymer is
approximately 60% materials and 40% fabrication (Ashby & Jones, 2012). Therefore, the focus on improving the fabrication techniques is to enhance the structural behaviour of the composites. Unfortunately, residual stresses are generated within the composites during manufacturing process (Safarabadi & Shokrieh, 2014). These stresses could diversely affect the structural behaviour of the fibre-reinforced composite. Residual stresses can happen in polymeric matrix composite due to the chemical shrinkage of matrix, the difference in the thermal expansion coefficients of the constituents, moisture content, and fibre prestress (Krishnamurthy, 2006). These stresses can develop within the matrix as a result of changing of the polymeric resin from liquid to solid phase. The mismatch of the thermal expansion coefficient between the reinforcement phase and the matrix phase can create undesired residual stresses in the composite structures when they are cooled down from their curing temperatures (Shokrieh, 2014; Krishnamurthy, 2006). Several methods have been used to minimise the detrimental effects of the induced residual stresses within the fibre reinforced composite such as optimisation of dwell cure cycle (White & Hahn, 1993b), curing the composite at low temperatures (Gopal et al., 2000) using electron beam curing (gamma irradiation) (Korenev, 2001), using expanding monomers (Fu et al., 2014), inserting shape memory alloy wires (Naghashian et al., 2014), and using fibre pretension method (Motahhari & Cameron, 1997).

Resin shrinkage and the difference in the thermal expansion coefficients of the composite’s constituents throughout the polymerisation and curing processes, can be considered as the main sources behind badly by-product residual stresses in the polymeric matrix composites (Metehri et al., 2009). The state of internal stresses of a laminated composite can impact its fatigue behaviour critically (Naik, 2003). Residual stresses in the aluminium layer of carbon fibre/epoxy–aluminium alloy and vinylon fibre/epoxy–aluminium alloy laminates have positive effects on their fatigue behaviours (Lin et al., 1991; Sui et al., 1996). Prestrain was applied in the laminates by the post-cured process to change the aluminium internal stress state from tensile to compressive before testing the laminate under fatigue loading. This process prolongs the laminate fatigue cycles-to-failure. Fibre pretension could be one of the available options for enhancing the mechanical properties of polymeric matrix composites (PMCs) without increasing their section dimensions or mass (Grażykowski et al., 2016). The pretension was applied to the fibre before and during the matrix curing. After the matrix was cured, the fibre pretension was released. Tuttle (1988) stated that fibre pretension could prolong the fatigue life of the fibre prestressed laminated composite compared with the non–prestressed counterparts. Krishnamurthy (2006) applied different levels of fibre prestressing in unidirectional E–glass/epoxy composite. The prolong in the tension–tension fatigue life increased to about 75% when applying a fibre prestressing level of 51 MPa and a normalised peak stress (a ratio of maximum varying tensile stress to the ultimate tensile strength of the composite) equalled 0.4. The reduction in the tensile residual stress within the matrix due to the fibre prestressing was the main reason behind this improvement of fatigue life. Harris (1977) identified that fatigue damage of fibre–reinforced plastics depended on matrix and interface strengths. Therefore, any process that can increase the matrix strength against crack propagation or fibre–matrix bonding might enhance the fatigue life of the composite as well. Talreja (2016) showed that the fatigue
damage in the low stress fatigue region consisted of matrix cracking and fibre–matrix interfacial debonding. Fatigue life extension in the low fatigue stress region was due to toughening of the matrix against cracking as a result of compressive residual stresses within it (Krishnamurthy, 2006). Previous studies confirmed the advantages of fibre prestressed composites for plain-weave composites (Mostafa et al., 2016a, 2016b, 2016c, 2017a, 2017b). Fibre pretension (prestressing) during matrix cure has generally a positive effect on the composite mechanical properties as it induces compressive residual stresses in the matrix. This study was inspired by the fact pretension on the composite’s strength for tension-tension fatigue loading has not been considered.

Therefore, the main objective of this study was to investigate the effect of biaxial fibre prestress on the fatigue life of the plain weave composite subjected to tension-tension cyclic loadings. In order to perform this task, the macro-mechanics lamination theory of the composite was developed in order to include the fibre prestressing effect and the resulting residual stresses within the composite’s constituents.

**MATERIALS AND METHOD**

The composite used in this study was E-glass plain weave fabric/polyester resin system. Table 1 lists the mechanical properties of the composite’s constituents.

Table 1

<table>
<thead>
<tr>
<th>Property</th>
<th>E-glass fabric</th>
<th>Polyester resin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus (E), GPa</td>
<td>70</td>
<td>2.77</td>
</tr>
<tr>
<td>Tensile strength (σ_{UTS}), MPa</td>
<td>2200</td>
<td>61</td>
</tr>
<tr>
<td>Poisson’s ratio (ν)</td>
<td>0.23</td>
<td>0.25</td>
</tr>
<tr>
<td>Thermal expansion coefficient (α), 10^{-6} m/(m°C)</td>
<td>120</td>
<td>5.4</td>
</tr>
<tr>
<td>Warp density × Fill density of the fabric (ends/m)</td>
<td>285 × 245</td>
<td>-</td>
</tr>
</tbody>
</table>

The analysis of the fibre reinforced composite was based on the classical lamination theory (CLT). The stress–strain (σ – ε) matrix was expressed by Jones (1999) as:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}
\]

[1]

The subscription 1 denotes the direction of the warp fibres and 2 as the direction of fill fibres.
The reduced stiffnesses $Q_{11}, Q_{22}, Q_{12}$ and $Q_{66}$ was:

$$
\begin{align*}
Q_{11} &= \frac{E_1}{1-\nu_{12}\nu_{21}} \\
Q_{22} &= \frac{E_2}{1-\nu_{12}\nu_{21}} \\
Q_{12} &= \frac{\nu_{12}E_1}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} \\
Q_{66} &= G_{12}
\end{align*}
$$

where

$G_{12}$ : Shear modulus of a composite.

$E_1$ : Elastic modules of a composite along the direction of fibres

$E_2$ : Elastic modules of a composite transverse to the direction of fibres

$\nu_{12}$ : The strain in the transverse direction over the strain in the direction of fibre when the stress is applied along the fibre’s direction

$\nu_{21}$ : The strain in the direction of fibre over the strain in the transverse direction when the stress is applied along the transverse direction

Residual stress induced in the composite due to chemical shrinkage of the resin is relatively less compared to that of thermal variation (White & Hahn, 1993a). Therefore, the developed residual stress–strain relation of a laminated composite due to thermal variation and fibre prestress could be rewritten as:

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12\text{ res}}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12\text{ res}}
\end{bmatrix}
$$

$$
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12\text{ res}}
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_1^{\text{pre}} - \varepsilon_1^{\text{ther}} \\
\varepsilon_2^{\text{pre}} - \varepsilon_2^{\text{ther}} \\
0
\end{bmatrix}
$$

When curing the composite at a temperature different from the ambient temperature, thermal strain was generated within the composite’s constituents. This strain was expressed by:

$$
(\varepsilon_{1,2\text{ res}}^{\text{ther}}) = \alpha_{1,2} \Delta T
$$

where

$\alpha_{1,2}$ : Coefficients of thermal deformation in the 1 and 2 directions, respectively

$\Delta T$ : Difference between the cure and cool-down temperatures
Prestressed Composite under Fatigue Loading

In the case of pretensioning the plain–weave fabric in the biaxial directions (i.e. the warp and the fill yarns), the induced residual strains $\varepsilon_1$ and $\varepsilon_2$ due to releasing the fibre pretension load after the matrix was cured can be expressed as:

$$
(\varepsilon_1^{\text{pre}})^{\text{res}} = \frac{(a^{\text{pre}}V_f)_{1-\text{dir}}^{\text{1}}}{E_1} - \frac{\nu_{12}(a^{\text{pre}}V_f)_{2-\text{dir}}^{\text{2}}}{E_1}
$$

$$
(\varepsilon_2^{\text{pre}})^{\text{res}} = \frac{(a^{\text{pre}}V_f)_{2-\text{dir}}^{\text{2}}}{E_2} - \frac{\nu_{21}(a^{\text{pre}}V_f)_{1-\text{dir}}^{\text{1}}}{E_2}
$$

where

$E_1, E_2$: Effective elastic modulus in warp and fill directions, respectively

$\nu_{12}, \nu_{21}$: Effective Poisson’s ratio

$V_f$: Fibre volume fraction

The fibre and matrix are denoted by the subscriptions $f$ and $m$ respectively. At the micro–mechanical level, the strain in both the fibre and the matrix was equal to the lamina strain due to compatibility of strain (Mostafa et al., 2017a). Residual stresses in the fibre ($\sigma_f^{\text{res}}$) and the matrix ($\sigma_m^{\text{res}}$) at a ply level can be calculated using equations 8 and 9:

$$
\sigma_f^{\text{res}} = E_f (\varepsilon_1^{\text{pre}} - \varepsilon_1^{\text{ther}})^{\text{res}} + \sigma^{\text{pre}}
$$

$$
\sigma_m^{\text{res}} = E_m (\varepsilon_1^{\text{pre}} - \varepsilon_1^{\text{ther}})^{\text{res}}
$$

According to Gay’s approach (Gay, 2015), the equivalent elastic properties were equal to:

$$
\begin{align*}
\tilde{E}_1 & \approx \beta E_1 + (1 - \beta)E_2 \\
\tilde{E}_2 & \approx (1 - \beta)E_1 + \beta E_2 \\
\tilde{G}_{12} & = \tilde{G}_{12} \\
\tilde{\nu}_{12} & \approx \frac{\nu_{12}}{(\beta + (1 - \beta)\frac{E_1}{E_2})} \\
\tilde{\nu}_{21} & \approx \tilde{\nu}_{12} \frac{E_2}{E_1}
\end{align*}
$$

where

$$
\beta = \frac{n_1}{n_1 + n_2}
$$

Here $n_1$ and $n_2$ denote the number of yarns per meter along the warp and fill directions that are equal to 285 and 245 respectively (see Table 1).
The effective composite thermoelastic properties of a unidirectional composite according to the concentric cylinder approach are equal to (Mostafa et al., 2017a):

$$E_1 = E_f V_f + E_m V_m + \frac{4(v_f - v_m)^2 V_f V_m}{V_m/\kappa F + V_f/\kappa M + 1/G_m}$$

$$E_2 = \frac{2(1-v_f) V_f - v_m V_m)kE_k}{(E_f + 4kV_f)}$$

$$G_{12} = \frac{G_m (G_{f,m}V_m + G_f (1 + V_f))}{(G_f V_f + G_m (1 + V_f))}$$

$$G_{m,f} = \frac{E_m f}{2(1 + v_m f)}$$

[12a]

and

$$v_{12} = v_f V_f + v_m V_m + \frac{(v_f - v_m) (1/\kappa M - 1/\kappa F)V_m V_f}{V_m/\kappa F + V_f/\kappa M + 1/G_m}$$

$$v_{21} = v_{12} \frac{E_2}{E_1}$$

$$\alpha_1 = \frac{\alpha_f E_f V_f + \alpha_m E_m V_m}{E_f V_f + E_m V_m}$$

$$\alpha_2 = \alpha_m V_m + \alpha_f V_f + \left(\frac{v_f E_m - v_m E_f}{E_m + E_f} \frac{V_f}{V_m}\right) (\alpha_f - \alpha_m)$$

[12b]

The axial residual stress $t$ induced in the lamina due to the biaxial fabric prestress and thermal residual stresses is calculated from equation 3:

$$\sigma_{1,\text{res}} = Q_{11} \varepsilon_{1,\text{res}} + Q_{12} \varepsilon_{2,\text{res}}$$

[13]

where $Q_{11}$ and $Q_{12}$ in the above equation were determined from equation 2 that related the elastic properties of the composite’s constituents with the fibre orientation angle; however, the $\varepsilon_{1,\text{res}}$ and $\varepsilon_{2,\text{res}}$ are the residual strains obtained from equations (3 and 4). Now, if the composite lamina is subjected to external axial tensile stress $\sigma_{\text{axi}}$ in the direction of specimen length, the total axial stress $\sigma_{\text{total}}^{\text{total}}$ within the composite lamina at that direction would become:

$$\left(\sigma_{\text{axi}}\right)_{\text{total}} = \sigma_{\text{axi}} + \sigma_{1,\text{res}}$$

[14]

The axial tensile stress in the composite resulted from applying external axial load without considering residual stresses is equal to:

$$\sigma_{\text{axi}} = \frac{p_{\text{axi}}}{A_{\text{total}}}$$

[15]
where $\sigma_{\text{axi}}$, $P_{\text{ax}}$, and $A_{\text{total}}$ represent the axial stress, axial load and the cross-sectional area of the composite material respectively. Consequently, by substituting equation 15 into equation 14, one can express the total internal axial stress in the prestressed composite as:

$$\left(\sigma_{\text{axi}}\right)_{\text{total}} = \frac{P_{\text{ax}}}{A_{\text{total}}} + \sigma_{1}^{\text{res}} \quad [16]$$

The relationship of stress–cycle ($\sigma$-$N$) fatigue data is usually defined by the power law equation according to Wöhler diagram. The stress-cycle relationship is expressed by:

$$\sigma_{\text{max}} = a N_f^b \quad [17]$$

where $\sigma_{\text{max}}$ is the maximum applied cyclic stress, $N_f$ is the number of cycles to failure. The symbols $a$ and $b$ are constants. Equation 17 is developed to include the effect of the fibre prestress, such that:

$$\left(\sigma_{\text{max}}\right)_{\text{total}} = \sigma_{\text{max}} + \sigma_{1}^{\text{res}} = a N_f^b \quad [18]$$

or

$$\left(\sigma_{\text{max}}\right)_{\text{total}} = a N_f^b - \sigma_{1}^{\text{res}} \quad [19]$$

The constant $a$ is equal to the ultimate tensile strength of the composite lamina ($\sigma_{\text{UTS}}$).

$$\sigma_{\text{max}} = \sigma_{\text{UTS}} N_f^b \quad [20]$$

**RESULTS AND DISCUSSION**

The tensile residual stresses in the fabric when it was prestressed with different levels and released after matrix cure are listed in Table 2. It is clear that increasing the biaxial fibre prestressing level could increase both the tensile residual stress in the fibre and the compressive residual stress within the matrix. On the other hand, the induced residual stresses within the matrix at warp and fill directions of the fabric were not equal due to using unequal yarn number per meter along the principal directions of the used E-glass fabric.

<table>
<thead>
<tr>
<th>Biaxial fabric prestressing (MPa)</th>
<th>Tensile residual stress in the fibre (MPa)</th>
<th>Residual stress in the matrix (warp direction) (MPa)</th>
<th>Residual stress in the matrix (fill direction) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>5.841</td>
<td>-1.14</td>
<td>-1.26</td>
</tr>
<tr>
<td>50</td>
<td>11.68</td>
<td>-2.27</td>
<td>-2.53</td>
</tr>
<tr>
<td>75</td>
<td>17.52</td>
<td>-3.39</td>
<td>-3.75</td>
</tr>
<tr>
<td>100</td>
<td>23.30</td>
<td>-4.52</td>
<td>-5.00</td>
</tr>
</tbody>
</table>
The axial stress in the lamina results from applying axial tensile stress that was developed in equation 14 is shown in Figure 1. The fibre volume fraction of the composite lamina was equal to 11% with cross-sectional area of 3 mm (thickness) × 25 mm (width). This value of fibre volume fraction was considered in previous studies (Mostafa et al., 2016a, 2016b, 2016c, 2017a, 2017b). The effect of the fibre prestressing was very clear in reducing the tensile stresses in the prestressed lamina when subjected to external axial tensile stress.

In general, the tensile stress in the prestressed composite lamina could be reduced by increasing the biaxial fabric prestress (Mostafa et al., 2016a, 2016b). This reduction continues as prestressing level increases, but this behaviour does not always result in improving the mechanical properties of the composite as seen in many experimental studies (Krishnamurthy, 2006). The main reason behind this is the existence of interfacial shearing stress between the fibre and matrix (Mostafa et al., 2017b), which is not considered in the classical lamination theory.

Table 3 shows the results obtained from using equation 14 and the experimental data of the unidirectional prestressed E-glass/epoxy composite obtained by Hadi & Ashton, (1998). Elastic moduli of the used composite’s constituents were 72.5 GPa and 3.45 GPa for the E-glass fibre and epoxy resin, respectively. The improvement in the tensile strength of the fibre prestressed composite due to applying different fibre prestressing levels is also reported by Hadi and Ashton (1998) for the composite fabricated with two different fibre volume factions (i.e. 35% and 60%). Table 4 shows the experimental data obtained by Mostafa et al. (2016a) and the current study results for a plain-weave E-glass/polyester composite system with two different equi-biaxial fabric prestressing levels. The ultimate tensile strength and the critical stress of the non-prestressed composite samples were equal to 66.5 MPa and 38.21 MPa (Mostafa et al., 2016a) respectively. Here, the critical stress represents the magnitude of stress at which the composite sample exhibits the first obvious sign of matrix fracture (onset
of failure). The findings of this study are supported by Mostafa et al. (2016a). The results of the developed fatigue equation were also compared with the available fatigue data obtained by previous studies. Fatigue data (cycles to failure) obtained by Krishnamurthy (2006) of the unidirectional E-glass/epoxy composite system with fibre volume fraction equals to 60% are listed in Table 5 along with the results of the developed theoretical fatigue equation. According Krishnamurthy (2006), the ultimate tensile strength ($\sigma_{UTS}$) of the used composite system and the fibre prestressing level was 1311 MPa and 51 MPa respectively. Different normalised peak stresses are used to validate the current results with Krishnamurthy’s experimental data. The comparison revealed an acceptable convergence between the theoretical results and the experimental data with a maximum absolute error at 11.5%.

Table 3
*Experimental tensile strength versus current theoretical results of a unidirectional e-glass/epoxy composite with different fibre prestressing levels*

<table>
<thead>
<tr>
<th>Fibre prestressing level (MPa)</th>
<th>Theoretical (current study) (MPa)</th>
<th>Experimental (Hadi &amp; Ashton, 1998) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_f$%</td>
<td>$V_f$%</td>
</tr>
<tr>
<td>35</td>
<td>60</td>
<td>35</td>
</tr>
<tr>
<td>25</td>
<td>401</td>
<td>404±8.2</td>
</tr>
<tr>
<td>50</td>
<td>408</td>
<td>419±6.1</td>
</tr>
<tr>
<td>75</td>
<td>416</td>
<td>422±7.2</td>
</tr>
<tr>
<td>100</td>
<td>425</td>
<td>451±8.3</td>
</tr>
<tr>
<td>200</td>
<td>460</td>
<td>468±8.7</td>
</tr>
</tbody>
</table>

Table 4
*Experimental tensile strength and critical stress versus current theoretical results of a plain-weave e-glass/polyester composite with different equi-biaxial fabric prestressing levels*

<table>
<thead>
<tr>
<th>Fibre prestressing level (MPa)</th>
<th>Theoretical (current study)</th>
<th>Experimental (Mostafa et al., 2016a)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ultimate tensile strength</td>
<td>Critical stress</td>
</tr>
<tr>
<td>25</td>
<td>70.5</td>
<td>42.21</td>
</tr>
<tr>
<td>50</td>
<td>74.5</td>
<td>46.21</td>
</tr>
</tbody>
</table>

Table 5
*Comparison of unidirectional e-glass/epoxy composite fatigue cycles to failure with different normalised peak stresses*

<table>
<thead>
<tr>
<th>Normalised peak stress $[(\sigma_{max})<em>{total}/\sigma</em>{UTS}]$</th>
<th>Cycles to failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theoretical (current study)</td>
</tr>
<tr>
<td>0.40</td>
<td>1,260,716</td>
</tr>
<tr>
<td>0.45</td>
<td>436,434</td>
</tr>
<tr>
<td>0.50</td>
<td>109,544</td>
</tr>
</tbody>
</table>
The developed fatigue equation of the prestressed composite (equation 19) is plotted in Figure 2. For a given applied maximum fatigue stresses ($\sigma_{\text{max}}$), the fatigue cycles to failure ($N_f$) were investigated. For the case of comparing the results of the prestressed composites prestressed in the presence of different levels of biaxial fabric prestress, (a) and (b) could be assumed constants for the same composite material. As an example, (a) and (b) were assumed equal to 35 MPa (Mostafa et al., 2016a) and -0.2, respectively. The maximum applied fatigue stress is taken to be 15 MPa. The fatigue cycles to failure were increased by 36 times than the non-prestressed counterpart as the prestressing level was increased to 100 MPa. The prolong in the fatigue cycles to failure is caused by reducing the tensile residual stress induced in the composite lamina by thermal cooling down from 50°C to 25°C during the manufacturing process.

In general, the fatigue cycles to failure of the prestressed composite lamina is improved by increasing the biaxial fabric prestress. This improvement continues as the prestressing level increases. This behaviour is mainly due to increasing the compressive residual stress within the matrix; therefore, more cycles are required in order to start the damage in the composite.

**CONCLUSION**

The biaxial fibre prestress of the composite reinforced with plain weave fabric reduced the tensile residual stresses within the matrix of the composite lamina that resulted from different thermo–mechanical properties of the composite’s constitutions. The tensile stress in the fibre prestressed composite lamina reduced by increasing the level of the biaxial fibre prestress as a result of compressive residual stresses within the lamina. This could improve the internal residual stress state of the composite subjected to tension-tension fatigue loading.
REFERENCES


