

Action Research in Examining the Enquiry Approach of Lesson Study in Mathematics

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ABSTRACT

Action Research (AR) focusses on classroom research for improving instructional practices while the Japanese inquiry-model of Lesson Study (LS) emphasises on teachers' collaborative work on student-engaged lessons. Both approaches are central to professional development as they draw on students' feedback for designing quality lessons. Through AR design, this paper reports on the effectiveness of LS in enhancing the enquiry approach and student engagement in a mathematics classroom. The main objective is to investigate and demonstrate the value and use of enquiry approach and how teachers can engage students in the development of the concept of perimeter during problem-solving, which was investigated by analysing the type of questions asked and the follow-up instructional activities. Our data focus on questions the teacher posed, classroom observation of 27 Grade Four students' behaviour while solving a perimeter problem, and teacher's feedback during the debriefing session. Findings revealed higher use of open-ended questions

(88.89%) that elicited higher-order thinking than closed questions (11.11%). We argue that the enquiry approach in this LS probed students' ability to explore alternative solutions, extend their prior conceptual understandings, developed teacher's support for activation, and use of higher-order mathematical thinking during the course of searching for solutions. Results suggest that LS constitutes a powerful context to understand the quality of questions

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that teachers use during their day-to-day practice. Further, we have evidenced that LS conducted within an AR design could be a useful research strategy in better understanding teachers' support for the development of 21st Century skills among our students.

Keywords: Action research, higher-order thinking, lesson study, mathematical communication and thinking, open-ended questions

INTRODUCTION

What is an ideal mathematics classroom?

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction ... The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding ... Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it. (National Council of Teachers of Mathematics [NCTM], 2000)

The following extract outlines the mathematical process of teaching and learning that should occur in the 21st Century mathematics classroom. It is

expected that through teacher-designed activities that engage students during problem-solving, students will actively participate in constructing new knowledge through discovery. And as teachers facilitate students who are engaged in mathematical communication, the students' mathematical thinking will be further enhanced, steering way for meaningful learning to occur. Central to capturing the essence of meaningful learning in Mathematics is the enquiry or investigative approach, which is a salient element in the Japanese model of Lesson Study. Since AR and Lesson Study are frequently treated as separate entities, this study was conceived to show how the two can be integrated to reap the aggregated benefits of the marriage between them

Lesson Study and Action Research

“Research lessons help you see your teaching from various points of view... A lesson is like a swiftly flowing river; when you're teaching you must make judgments instantly. When you do a research lesson, your colleagues write down your words and the students' words. Your real profile as a teacher is revealed to you for the first time”. (Lewis & Tsuchida, 1998)

If teachers want to make changes in their teaching and student learning, they need to be provided with the opportunity to reflect on the choices they had made during the course of their classroom practice. That sets the need to embed lesson study in classroom AR. While AR answers specific classroom research questions,

lesson study also allows the AR to be done collaboratively. Furthermore, teachers gain more by participating in lesson study while conducting AR as they share and talk more about teaching, lesson, and resources under the mentorship of the ‘Knowledgeable Other’ (Takahashi, 2014).

The lesson study approach in classroom AR also creates many “eyes to see children” (Lewis, 2002b), which becomes sound testimony of teachers’ actions and concomitant student learning. Likewise, AR is a case study, defined by Merriam (1989) as “an intensive, holistic description and analysis of a single entity, phenomenon, or social unit.” The unit within the context of the study is most often a classroom of students. Activating the two approaches in tandem, Lesson study, which creates “many eyes to see children” complements AR that provides a “holistic description” of particular instructional practices in a way that researchers’ feedback of students’ learning, students’ feedback and discussions with the ‘Knowledgeable Other’ form the basis of improving the lesson. Thus, lesson study in researching classroom issues (by using AR) provides solutions on how to improve learning, which is not possible through formative or summative assessments that only provide information on what to improve, without addressing the crucial *how*. As such, lesson study can be a powerful approach in answering research questions when conducting classroom AR in the 21st century (Lewis, 2002a).

Accordingly, this study was conceived by combining AR and the inquiry element

of the Japanese Lesson Study approach in researching the effectiveness of using Lesson Study in a mathematics classroom. Therefore, this study is focussed on investigating the nature of inquiry approach in Lesson Study by examining the type of questions teacher asked and the follow-up activities that the teacher provided, which will help to explain students’ mathematical thinking processes and answers the research question ‘How can teachers engage students in the development of the mathematical concept of perimeter through problem-solving?’

Mathematical Thinking

The most acceptable understanding of mathematical thinking is as a mental process that includes the presence of at least one mathematically-related activities during the process of problem-solving such as reasoning, analysing, synthesising, abstraction, symbolic representation, symbolic manipulation (Schoenfeld, 1992), understanding mathematical ideas, establishing relationships among the mathematical ideas, solving the problem (Lutfiyya, 1998), and proofing (Harel et al., 2006). In particular, Stacey (2007) posited that deep mathematical knowledge, general reasoning abilities, and knowledge of heuristic strategies were essential elements of mathematical thinking. Another perspective refers to mathematical thinking as exploring the world and communicating about it by ‘doing mathematics’ or mathematising (Romberg & Kaput, 1999). Accordingly, mathematical thinking occurs during

problem-solving activities because students are engaged in a meaningful setting usually through communication. As Romberg and Kaput (1999) explained:

Curriculum activities that reflect this perspective are those that involve students in problem-solving and that encourage mathematization. In addition, they encourage the use of mathematical languages for expressing, communicating, reasoning, computing, abstracting, generalizing, and formalizing. These systems of signs and symbols extend the limited powers of the human mind in many directions, and they make possible a long-term (cross-generational) cultural growth of the subject matter. Finally, such situations embody systematic forms of reasoning and argument

In the wake of emphasising communication during mathematics problem solving, the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000) regards communication, problem-solving, reasoning and proof, connections and representation as essential components of any mathematics lesson as outlined in its Process Standards. Mathematical communication involves applying the correct mathematical notation, in addition to the correct use of mathematical language, symbols, and graphical representations. By communicating mathematically during problem-solving, students are able to argue

logically as they analyse mathematical ideas critically. NCTM (2000) reiterated the value of mathematical communication as an integral component of problem-solving both in a mathematics classroom and in real-life situations, for the construction process of new mathematical knowledge will fail to occur without students communicating mathematically. As students communicate mathematically, their mathematical ideas become clearer and they are also able to justify their mathematical thinking. It is only during problem-solving that students experience them. The fourth emphasis, Learning to communicate mathematically of the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1999), states that

The development of a student's power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking. (NCTM, 1989)

Mathematical reasoning is also developed during problem-solving when students are guided to make and prove conjectures, communicate mathematically to provide logical explanations and make analysis and justifications. During mathematical communication as well,

students master their ability to make representations when they establish relationships between real-world setting and the world of mathematics using multiple representations.

Quantitative literacy which embodies problem-solving, reasoning, and real-world applications (Roohr et al., 2014) is also another term that is associated with mathematical thinking. Since mathematical thinking is a broad term that encompasses multiple meanings and the thinking process occurring in students' minds is not transparent. Watson and Geest (2005) listed a series of activities that students needed to participate to show that mathematical thinking was in progress. Among them are

“choosing appropriate techniques, generating own enquiry, describing connections with prior knowledge, giving reasons, finding underlying similarities or differences, working on extended tasks over time, generalising structure from diagrams or examples, creating and sharing own methods, making comparisons, changing their minds, posing own questions, initiating their own mathematics”.

Therefore, it is rather difficult to single out one activity as they complement one another in enhancing mathematical thinking during problem-solving activities. Since mathematical thinking is an abstract process that occurs in one's mind and is not visible, assessing the type of mathematical thinking that is occurring can be achieved through the type of questions that teachers pose.

This is because when students respond, they rely on different types of thinking in answering those questions as discussed in the subsequent section.

Higher-order Thinking Skills and Lower-order Thinking Skills Questions

The importance of questioning students cannot be underestimated. As Reinhart (2000) stressed,

“Never say anything a kid can say! This one goal keeps me focused. Although I do not think that I have ever met this goal completely in any one day or even in a given class period, it has forced me to develop and improve my questioning skills. It also sends a message to students that their participation is essential. Every time I am tempted to tell students something, I try to ask a question instead”.

When students are posed with questions, they resort to various types of thinking to access the required and relevant mathematical knowledge to answer the question. Rashid and Qaisar, (2016) reiterated that the questions posed during a lesson influenced the type of thinking skills that was occurring. In particular, the type of oral questions posed to play a significant role in the development of thinking skills during problem-solving (Sprague, 2008).

According to Capacity Building Series (2011), the type of questions asked can be categorised as open questions and closed questions or commonly known as a ‘yes/no’ question. A closed question such as

“What is 4 + 6?” is contrasted with an open question such as “Is there another way to make 10?” (Capacity Building Series, 2011). Open questions help teachers build students’ self-confidence as they allow learners to respond at their own stage of development. Open questions intrinsically allow for differentiation of learning ability. Responses will reveal individual differences, which may be due to different levels of understanding or readiness, the strategies to which the students have been exposed, and how each student approaches problems in general. Open questions signal to students that a range of responses are expected and, more importantly, valued. By contrast, yes/no questions tend to stunt communication and do not provide useful information (Capacity Building Series, 2011).

Illuminating further, open questions are recognised for being effective for learning to occur as they motivate a plethora of responses and encourage students to think ‘out of the box’, which is central to higher-order thinking (HOT). Since the nature of open questions requires students to justify their response, students usually dwell in elaborative thinking (Lee et al., 2012), thus stimulating meaningful negotiations to occur, which most likely lead to the extended conversation (Maftoon & Rezaie, 2013). Therefore, open questions are associated with the higher levels’ of Bloom taxonomy such as reasoning and judgment (Hargreaves, 1984), and engage children in higher-order thinking (Roth, 1996). Subsequently, open questions promote students to process and reflect on their

thoughts and ideas, which eventually sets of self-regulated thinking skills (Zimmerman, 2013).

Since HOT include elaborative thinking skills such as critical thinking, creative thinking, deductive thinking, inductive thinking, rational thinking, analogical thinking, metaphor, metacognition, making inferences, making generalisation, making a conclusion, judging idea, making predictions, solving problems, analysing, proposing solutions, and comparing and hypothesising (Rajendran, 2010), open questions invoke students’ HOT. Accordingly, open questions elicit higher-order thinking (HOT) while closed questions confine the nature of thinking to lower-order thinking (LOT) as students’ responses are limited to a yes or no.

Mainali (2012) provided an insight into how HOT was related to the activities that students carried out in the classroom.

Students are engaged in HOT when they: visualize a problem by diagramming it, separate relevant from irrelevant information in a word problem, seek reasons and causes, justify solutions, see more than one side of a problem with sources of information based on their credibility, reveal assumptions in reasoning and identify bias or logical inconsistencies.

As such, the students’ responses to these verbal open questions provide insights into

their mathematical thinking that is involved and is captured through the mathematical communication that transpires during the inquiry-based mathematics lesson. Crucial to the type of thinking that is occurring within students is the type of questions that teachers pose (Capacity Building Series, 2011). This is because “the type of questions teachers ask is critical in providing the correct support in consolidating students mathematical knowledge as it shapes their mathematical thinking” (Capacity Building Series, 2011) and in “helping students to identify thinking processes, to see the connections between ideas and to build new understanding as they work their way to a solution that makes sense to them (Capacity Building Series, 2011).

Despite the many benefits of an open question, it is nevertheless not well received by teachers. Repeated studies conducted over many decades reveal that teachers use a higher proportion of closed questions than open questions (Brock, 1986; Galton et al., 1999; Lefstein & Snell, 2011; Maftoon & Rezaie, 2013) in their classroom. Of interest is the study by Brock (1986) who discovered that with proper training, teachers could be guided to ask more open questions (173) than closed questions (21). In comparison, untrained teachers tended to use a higher rate of closed (117) than open questions (24).

Therefore, in this study, the questions that the teacher posed during a mathematics inquiry-based lesson were recorded and analysed to determine whether they are open

or closed questions as open questions invoke HOT and closed questions invoke LOT.

THEORETICAL FRAMEWORK

Bruner’s constructivist theory articulates that students learn best when they are engaged in active inquiry, which allows them to “go beyond the information given” (Bruner, 1973). For meaningful and effective learning to occur, students need to individually interpret the information in reality based on their past learning experience. Accordingly, teachers need to create opportunities for them to make discoveries in the learning by designing appropriate tasks in the lesson that builds on their prior learning, leading to their anticipated current learning and also by engaging them in active dialogues.

RESEARCH DESIGN

This study was an attempt to use AR design in a Lesson Study context as a possible strategy for analysing teachers’ support for students’ to develop 21st Century skills, in particular, higher-order thinking skills and providing an innovative model of continuous professional development for mathematics teachers. This study was conceived by combining AR and the enquiry element of the Japanese Lesson Study approach in researching the effectiveness of using Lesson Study in a mathematics classroom. The lesson study approach involved collaborating with teachers to plan, observe, and reflect on lessons (Lewis, 2002b). An explorative AR design was conducted by designing

activities, implementing and observing two lessons, which were improved through debriefing sessions in two different cycles. This design allows teacher-researchers to be engaged in emerging and current classroom issues arising in their classrooms, which require urgent and immediate solutions. Both lessons were on the same topic and lesson objective of introducing the concept of perimeter conducted by two different teachers in two different schools, who were part of the Lesson Study team and AR group. Four members of the Lesson Study team observed and evaluated two different AR lesson cycles. The findings of the AR in the first lesson design were in the form of suggestions for improvement for the implementation of the second improved lesson. This research paper reports only on the findings of the AR on the second improved lesson of the Lesson Study cycle.

The researcher was an observer. Key components included training teachers on the Lesson Study and problem-solving activities, field observation of the participating classroom during implementation, and debriefing sessions with the teacher after implementation. The research project spanned a year. Data collection included on-line support during lesson planning and development for the teacher, field observation of the participating classroom (teacher and students) during lesson implementing, recording of the questions posed and analysis of the lesson plans, and the videotaped lessons.

This study was focussed on investigating the nature of inquiry approach in Lesson

Study by examining the type of questions teacher asked and the follow-up activities that the teacher provided, which would help to explain students' mathematical thinking processes and answers the research question 'How can teachers engage students in the development of a mathematical concept through problem-solving?'. In this study, mathematical thinking is operationally defined as occurring when there is evidence of students engaged in at least one activity of solving problems, communicating mathematically, and making mathematical reasoning (Ministry of Education Malaysia [MOE], 2012). Therefore, to substantiate the nature of mathematical thinking, the mathematical communication that transpired between the teacher and the students was further studied by quantitatively and qualitatively analysing the questions posed by the teachers to the class. These questions were recorded and categorised into open and closed questions as open questions invoke higher-order thinking (HOT) while closed questions invoke lower-order thinking (LOT).

METHODS

Prior to preparing an inquiry-based lesson, the teacher attended a one-day workshop on Inquiry Based Mathematics Education (IBME) and lesson study. The primary aim of the workshop was to introduce the principles of implementing Lesson Study and as a follow-up to the workshop, the participants would collaborate with the researchers to discuss and develop a lesson plan. Four schools participated and at least

four teachers from each school attended the workshop. The schools and the teachers participated out of their own free will in the workshop as they wanted to improve further their pedagogical knowledge in an attempt to move away from traditional classroom teaching. This set the baseline for this study. The participants were motivated to make their lesson more interesting, more creative, and simultaneously upskill their pedagogical content knowledge. Upon returning to the school, the participating teacher prepared a lesson plan that adopted inquiry-based pedagogy using online communication with the researchers.

The participating teacher and her team of two teachers prepared a lesson plan that was developed together with the team of three researchers, who formed the Lesson Study and AR group members. The lesson plan went through a series of revisions based on the feedback provided by the team members and the 'Knowledgeable Other'. The lesson was on the introductory concept of perimeter. The lesson was planned for one hour and began on time. The instructional steps were carried out in accordance with the lesson plan, with good time management. The students were 29 multi-ethnic high-achievers from a Grade 4 class in an urban co-educational school in Penang Malaysia. They were seated facing each other in groups of five. The lesson was videotaped and qualitative data was collected mainly by observing the students, their worksheets, and feedback from the teacher Lesson Study and AR group members through their field notes.

We report on mathematical thinking skills that students demonstrated when they were solving problems during the lesson. Therefore, evidence of events that occurred during the mathematics lessons to show how the teachers engaged students in the development of the concept and how mathematical thinking was enhanced due to the enquiry approach on Lesson study was cited. The focus is on mathematical communications, representation, and reasoning because of the themes that emerged from the data.

RESULTS

Problem Solving and Mathematical Skills in the Follow-up Activities

The lesson began with a problem posed to the students. The problem was about elephants intruding a coconut plantation. The teacher adopted the technique of story-telling simultaneously with a visual representation of cut-outs of elephants made from manila cardboards 'walking' into a plantation. That approach got the students laughing and interested. They were seen communicating with each other and also to the teacher on the other directions that elephants could have invaded the plantations, which got the teacher to get some students to help her 'walk' the elephants in. This indicates that students were able to make relations to the real-world and the world of mathematics. They also analysed and were reasoning mathematically by arguing the single direction the elephants were invading, which suggests evidence of mathematical thinking.

Inviting the students to help the teacher enhanced students' active participation and empowered them in the learning process.

The students were then posed with a real-life problem on how to prevent the elephants from invading the plantation. Despite multiple logical responses like 'scare them with loud noises' to even 'gunning them down', majority agreed on erecting a barbed-wire fence. To this, the teacher called one student at a time to paste the cut-out fences at the plantation to prevent the invasion. The students 'neatly' placed one fence beside another all around the plantation.

The main activity of the lesson developmental stage required the students to use a marker pen and multi-coloured papers. They were requested to cut the coloured papers into any two-dimensional geometric shapes and paste them to design another two-dimensional geometric figure with the condition that the adjoined geometric shapes must be of different colours. This activity was conducted in small groups of four to five students. The geometrical shapes that students created took multiple shapes and sizes. This suggests that students' creativity was harnessed in the lesson and was not stumped by providing pre-made geometrical shapes. As they were drawing the geometric shapes, they were enhancing the skills of manipulating instruments like rulers and they could be seen communicating and debating about the best shape that ought to be constructed as many students had many versions of how the final product should look like. In addition, as they were cutting

the geometric shapes, they were enhancing their mathematical skills of using scissors to cut polygons like square, rectangle, and triangle and exercising precision in cutting out the geometric shapes. While they were collaborating as a team to accomplish their tasks, the teacher moved around the classroom, gauging students' understanding and providing prompts when necessary.

After creating the two-dimensional geometrical shapes, the students were asked to use marker pens to outline the perimeter for that geometric figure. Their challenge was in drawing the outline that marked the perimeter. There was a group that drew the outline to the interior sides of the various geometrical shapes that created the single geometrical figure. Another group had 'gaps' in the exterior outline, which contradicted the concept of perimeter as a continuous line bordering a closed geometric figure.

Mathematical Communication, Representation and Reasoning

After the students had completed the group activities, one group member for each group was asked to present. It was during the presentation that students seemed very excited as they were eagerly pointing out how 'different' the geometric figures were. There was a lot of communication among themselves, to the teacher, and to the presenter in the form of questions, comments, and discussions, especially the group that presented on the perimeter with interior boundaries marked and with 'gaps' in the exterior outline. The teacher related to the earlier story of the plantation and the

students pointed out that the fencing was on the exterior boundaries of the plantation and the fencing must be securely adjoined to each other. Mathematical reasoning and logical thinking, relating, representation, and mathematical communication appear to have occurred. The students were able to make representations as the students provided answers that indicated that they were able to establish relationships between the real-world (plantation) and the world of mathematics (geometrical figure). They were able to relate the conceptual knowledge of perimeter being a continuous line bordering a closed geometric figure to the procedural knowledge of drawing the outline of the perimeter. At this juncture, it is necessary to highlight that the mathematical terminology of ‘perimeter’ was not introduced yet. The two activities were used to develop the concept of identifying the continuous line bordering a closed geometric figure.

After the presentations, the teacher introduced the mathematical register of the perimeter as the fencing that they had built at the beginning of the lesson and the outline of the marker pen in the second activity. The closure of the lesson involved the students explaining in their words the mathematical ideas on the concept of perimeter. Prompting from the teacher helped students to draw conclusions on what is not a perimeter based on the two groups’ misconceptions. The lesson ended with the teacher giving the students homework. Figure 1 exhibits the activities that occurred during the lesson.

Open and Closed Questions

A lot of questions were posed during the lesson. However, only questions related to the development of the conceptual understanding of perimeter were recorded as exhibited in Table 1. Rhetoric questions or questions requesting participation such



Figure 1. Activities during problem solving

as *Who can help to put the fence?* are not discussed in this section. From a total of 18 questions posed, 2 or 11.11% were yes/no (closed) questions, while 16 or 88.89 % were open questions that encouraged a variety of flexible responses from the students. In addition, there was the frequent use of *why*

and *how* during the lesson, mainly preceding students' responses. A total of five times, the teacher used the WH question words of *why* and *how* to seek clarification to students' responses. Table 1 exhibits the questions that were posed.

Table 1

Types of open and closed questions

Num	Open Question	Closed Question (Yes/No)
1	What does the picture show?	There is an opening here (pointing to the constructed fence). Is that acceptable?
2	What is happening now?	Your friend has drawn the inner boundary. Is that acceptable?
3	How to prevent the elephants from invading the plantation?	
4	Do you think we should kill them?	
5	With what?	
6	Any other way?	
7.	Discuss how you can construct the geometrical shape so that no adjacent cut-outs have the same colour	
8	The boundary drawn around your constructed geometrical shape has a gap. Why is this not acceptable?	
9.	Make a summary on what is not a perimeter	
10	What do you think is a perimeter? Use own words.	
11	Why when determining the perimeter, the interior boundary can not be included?	
12	What is not acceptable with the drawn boundary for this geometrical shape?	
13	In determining the perimeter, why there must not be a gap in exterior boundary?	
14	How do you determine the perimeter for this diagram?	
15	What did you learn today?	
16	In your words, describe perimeter?	

DISCUSSIONS

Problem-solving was the main activity in the lesson. Two problems were posed, with one real-word setting in the form of storytelling and the second problem posed during the lesson development, which was the main task that students had to complete as a group activity. The problem that was posed in the induction set was carefully designed to achieve the desired mathematical concept of perimeter. It also captured a real-life scenario. As students progressed through the process of solving the problem, the majority of the students acquired the concept, even though misconceptions occurred among two groups of students. Through the teachers' spontaneous facilitation and questioning, a reflective activity was conducted to address the misconception, where students made relations between the two activities that enhanced their understanding of perimeter as the continuous line bordering a closed geometric figure.

There was also clear evidence of mathematical communication occurring throughout the lesson as a result of questioning, specifically during the storytelling, group discussions, and group presentations. Communication refers to an interactive process that involves activities like speaking, listening, writing, and reading. It is one way to share mathematical ideas and clarify the understanding of Mathematics and can be in verbal or written modes (Cuevas, 1991). According to the National Research Council (1989),

..... research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: "examine," "represent," "transform," "solve," "apply," "prove," "communicate." This happens most readily when students work in groups, engage in discussions, make presentations, and in other ways take charge of their own learning.

In this lesson, oral communication most commonly occurred in the highest frequency among students to convey their mathematical thinking and it was a two-way interaction that transpired between teacher-student, student-student, and student-object. A high percentage of 99.89% of open questions and 11.11% of closed questions, preceded by the use of *why* seven times, indicate higher-order thinking occurring during the lesson.

During the group presentation, students' speaking skills using the mathematical language were enhanced when they were engaged in questioning the use of words such as *square*, *rectangle*, and *triangle*. Their mathematical ideas were reflected upon, discussed, and modified. Listening skills were developed as they listened to their peers explaining the mathematical ideas and their understandings were developed as they responded to what they hear. As such, it encouraged them to think

mathematically. They also 'read' the graphic forms of the two-dimensional geometric figures to rationalise their thoughts and make interpretations about the perimeter. The written skills were observed when students outlined the boundaries that mark the perimeter.

According to Jacobs et al. (2005), four essential strategies scaffold mathematical communications: rich tasks, safe environments, students' explanations and justifications, and processing of ideas. A detailed analysis of the lesson reveals that all four elements were present. The rich task is defined as being open-ended and challenging. A safe environment refers to a non-threatening environment that invites students to share their ideas, despite producing incorrect solutions. The emphasis is placed more on their ability to reason and make justifications, instead of the correct answer. The third strategy of making 'explanations and justifications' encourages students to 'think out loud' in order to communicate their mathematical ideas, while the final strategy refers to listening to the thoughts of others, which are manifestations of their thinking (Jacobs et al., 2005).

In the observed lesson, the task was rich as there were multiples correct solutions and was appropriate for the students. The environment was non-threatening as students were comfortably presenting their ideas individually and publicly in the classroom. During the group presentation, students were motivated to explain and justify their answers, and at the end of

the lesson, they provided the lesson summary using their own words. When the teacher was enacting the story of the elephants invading the plantation and when other groups presented, the students listened and provided their views when their mathematical understanding was challenged. Along the line of thought of Jacobs et al. (2005), the observed lesson had successfully promoted mathematical communication in the lesson. However, students' confidence level in communicating was much higher when they were in their groups as compared to communicating ideas individually.

The observable mathematical communication was easily captured, unlike the mental processes. However, evidence of students explaining their mathematical ideas and disagreeing with their peers' views could only have happened with the occurrence of mathematical thinking through the mathematical processes of reasoning, relating, and representing. In addition, based on the work of Watson and Geest (2005), some of the activities that suggested mathematical thinking occurred in this lesson. Students were involved in selecting suitable techniques to create their preferred geometrical shapes. They were reasoning and were sharing their methods amidst comparing their work with their peers. They were also posing questions

During the debriefing session with the 'Knowledgeable Other', the teacher also shared her views on how much her students enjoyed the lesson and cited the vibrant atmosphere as a result of students' active

engagement in the activities. However, she also agreed that the lesson consumed time and effort as it involved careful planning and spontaneous facilitation, but was worth the effort and time invested in the lesson planning and implementation. She “felt more confident in handling the lesson. This is because of the meticulous preparation for this lesson, even though it took more time. But it was worth it.”

She also felt that the inquiry approach helped here to improve her skills in facilitation.

“ I felt that I have learned a lot about how to facilitate my students. Most of the time, we tell students what to do. But, during this and after this lesson, I learned the art of asking the correct questions to get students to think and seek solutions. I think that all the more made students enjoy the class more.”

She also felt that the class was less rigid and the students were having meaningful learning as they communicated the ideas freely.

“The activities during problem-solving enabled the students to respond freely and creatively in responding to the tasks. They also enjoyed moving to some groups to see their friends at work. They also had the opportunity to share their views with their other groupmates.”

The overwhelmingly positive feedback from the teacher neither meant that her past

lessons were not effective nor not enjoyed by her students. As she said, “the students enjoyed it more than usual and the lesson was more effective than the traditional way of teaching” and the type of questions that she asked probed her students to think deeply and more creatively than usual. Therefore, the teacher’s facilitation skills played a crucial role as well in making the lesson more meaningful through the mathematical processes that the students underwent.

These mathematical processes and skills were invoked among students as a result of the teacher’s facilitation skills as without her ability to facilitate, the mathematics lesson would have turned into a traditional class whereby the teacher executes the role of the knowledge disseminator and students passively receiving the knowledge. The lively atmosphere of the class throughout the duration of the lesson is a testament to her facilitation skills enhancing students’ mathematical thinking in a mathematics inquiry-based lesson.

CONCLUSION

Lessons created through lesson study tend to be an inquiry-based lesson as teachers work collaboratively in a small group with the ‘Knowledgeable Other’. As a result, in this study, students’ mathematical thinking was enhanced through the use of open questions during problem-solving and was demonstrated by mathematical communication, reasoning and representation during questioning. Their mathematical thinking was developed

through the use of open questions during the stages of solving the open-ended problem, students' self-learning through problem-solving activities, whole-class discussion, and summarising or explaining using their own words. Lesson study also promoted teacher facilitation.

From the findings, it is evident that students' higher-order mathematical thinking was enhanced due to the element of enquiry in Lesson Study. In addition, through problem-solving activities, the nature of enquiry was further augmented when students were able to communicate mathematically and do mathematical reasoning

It is also apparent that AR is becoming an important research design tool in addressing classroom pedagogical issues in a nuanced manner. Its versatility and power in generating valuable data about *in situ* practice are evident when incorporated into Lesson study, mainly because of the many indistinguishable common features shared by both approaches. Both are enquiry-based, with a focus on improving teachers' pedagogical content knowledge through collaboration. However, there are mixed views on which approach is more structured. While Ferrance (2000) claimed that AR was a highly structured discipline of inquiry, Dudley (2011) asserted that lesson study as a "highly specified form of classroom action research...". On the other hand, there is another third perspective, which views the two as parallel approaches. Lesson study when implemented as a type of classroom

practitioner research remodels into AR. This point of view emerged from their aligned methodologies, shared common foci (Lewis et al., 2009), and the use of feedback as authentic evidence of students' learning (Willis, 2007).

However, the extensive use of AR should not be confined to the classroom and this study has attempted to take an innovative approach in melding AR and Lesson Study to capture the enormous benefits that AR can offer. While the primary aim of this paper is to highlight the effectiveness of Lesson Study by studying the nature of the inquiry approach and student engagement through the type of questions asked and follow-up activities provided by the teacher in the classroom. This paper resonates with the perspective of moving away from treating AR and Lesson Study separately and integrate them as the way forward in conducting effective classroom research in the 21st Century.

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