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On Estimating the Parameters of the Generalised Gamma Distribution Based on the Modified Internal Rate of Return for Long-Term Investment Strategy

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ABSTRACT

The generalised gamma distribution (GGD) is one of the most widely used statistical distributions used extensively in several scientific and engineering application areas due to its high adaptability with the normal and exponential, lognormal distributions, among others. However, the estimation of the unknown parameters of the model is a challenging task. Many algorithms were developed for parameter estimation, but none can find the best solution. In this study, a simulated annealing (SA) algorithm is proposed for the assessment of effectiveness in determining the parameters for the GDD using modified internal rate of return (MIRR) data extracted from the financial report of the publicly traded Malaysian property companies for long term investment periods (2010–2019). The performance of the SA is compared to the moment method (MM) based on mean absolute error (MAE) and root mean squares errors (RMSE) based on the MIRR data set. The performance of this study reveals that the SA algorithm has a better estimate with the increases in sample size (long-term investment periods) compared to MM, which reveals a better estimate with a small sample size (short-time investment periods). The results show that the SA algorithm approach provides better estimates for GGD parameters based on the MIRR data set for the long-term investment period.

Keywords: Generalised gamma distribution, modified internal rate of return, moment methods, simulated annealing algorithm

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INTRODUCTION

Recently, one of the major concerns in the investment decision is stock assessment and forecasting. An investment decision is one of the vital activities in business performance.

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Both shareholders and investors can use stock valuation models to evaluate their shares and make stock trading decisions accordingly. Major shareholders carried out investment valuations to drive them to make decisions using various models. Managers must have a correct grasp of the influential resources to construct an investment model for stock valuation, which is a vital aspect of a company's performance in stock valuation.

Investment modelling is all about building and analysing mathematical models to depict the processes by which money flows into and out of a company. It takes various quantitative techniques to "make financial sense of the future." Investors may use a variety of models when selecting stocks and evaluating the performance of their stock portfolios. Stock valuation is a strategy used in determining the worth of a firm, which may provide information to potential investors about the company's profitability constraints (Besley & Brigham, 2015).

Stock valuations are widely conducted based on traditional accounting methods. It includes the Net Present Value (NPV) and the Internal Rate of Return (IRR), and the Profitability Index (PI) (Bonazzi & Iotti, 2016). The implementation of IRR calculation, in which a single investment project came with multiple values, is one of the major flaws of traditional accounting methods (Kierulff, 2008; Osborne, 2010; Sabri & Sarsour, 2019). This problem is overcome by the application of the MIRR, which was rediscovered in the 1950s after its development during the 18^{th} century, and which accounts for the periodic free cash flows by assuming reinvestment of cash inflows at the reinvestment rate (Baldwin, 1959; Biondi, 2006; Kierulff, 2008; Sabri & Sarsour, 2019).

The MIRR is defined as the rate at which the NPV equals zero, that is, where the Present Value (PV) of the investment fund's terminal value (future value of cash inflows assumed to be reinvested at the firm's required rate of return) equals the present value of the investment outlays (cash outflows over the investment period) when discounted at the firm's required rate of return (Besley & Brigham, 2015; Quiry et al., 2005). The investment rate of return might be accomplished using an iterative procedure to locate the root, such as the Newton-Raphson algorithm (Ahmad, 2015) or the modified Newton-Raphson method (Pascual et al., 2018).

However, some problems arise when utilising this procedure since it does not account for all relevant aspects that affect the investment return, rendering their methods ineffective for measuring stock performance (Brealey et al., 2006; Markowitz, 1952; Ross et al., 2010). As a result, several researchers have devised alternate investment appraisal methodologies to address this issue (Sabri & Sarsour, 2019; Satyasai, 2009; Sayed & Sabri, 2022). Sabri and Sarsour (2019) formulate an effective approach for determining the rate of return from long-term investments considering a variety of financial parameters, including stock price, reinvested dividends, and share issuances such as splits and bonuses issues. Investment returns were more realistically shown in the stock investment model since it revealed the calculation of the MIRR using a yearly annuity-style approach to contributions.

Sayed and Sabri (2022) extend the process of calculating the MIRR by incorporating treasury share dividends, which may significantly affect the investment's rate of return. Moreover, the study suggests transforming the MIRR by adding the value of one to prevent any negative return since the return estimated on any investment appears to be more than minus one. Being a random variable, the rate of return will follow some statistical distribution. The normal distribution in finance is often used to model asset returns. For example, Sharpe (1964) assumed that the return follows a normal distribution while describing the theory of market equilibrium and Capital Asset Pricing Models. However, returns on financial assets do not exhibit the normal distribution (Cont, 2001). As a result, other distributions should be employed (Fama, 1963).

The gamma distribution is extensively employed in several scientific and technical disciplines, such as banking, networking, and meteorological research (Kellison, 2009; Kim et al., 2003) due to its high degree of adaptability with the normal and exponential distributions, among others (Eric et al., 2021). This distribution is used to represent positive continuous variables. It logically models the waiting durations between events, like Sabri and Sarsours (2019), who modelled a framework with positive and continuous variables. A unique and flexible form of the gamma distribution is the GGD which includes special cases of some distributions such as the Weibull distribution, the gamma distribution, the exponential distribution, and the lognormal distribution (Kiche et al., 2019; Khodabina & Ahmadabadi, 2010; Stacy & Mihram, 1965).

Various approaches, such as the method of moment and maximum-likelihood function, exist to estimate the three parameters of the GGD, namely, shape, scale, and growth rate (Naji & Rasheed, 2019). Estimating the GGD parameters based on the numerical methods is problematic due to the difficulty in deriving their values from the mean and variance equations unless the value of one parameter is fixed and the values of the other two are calculated. Also, utilising the maximum-likelihood function to estimate the three parameters simultaneously is a mathematically complex procedure since it is difficult to derive a simple differentiation of the log-likelihood function concerning the shape parameter (Gomes et al., 2008; Lakshmi & Vaidyanathan, 2016; Özsoy et al., 2020). Hence, the SA algorithm, which uses the log-likelihood function as an objective function and seeks to maximise it, may be incorporated as an alternative parameter estimation method (Idris & Muhammad, 2022).

The SA algorithm attempts to produce novel solutions to a particular problem based on a random process and a series of probability distributions. This random procedure does not necessarily improve the objective function but may still be accepted (Franzin & Stützle, 2019). The algorithm was first employed in metallurgy as an optimisation procedure to attain minimal energy by progressively lowering atomic mobility, decreasing lattice defects' uniformity, and reducing metal temperature (Du & Swamy, 2016). It is unaffected by any restraint at any local minimum and accepts any changes in the objective function with indulgence, making it useful in many fields, including finance, mathematics and statistics (Abubakar & Sabri, 2021a, 2021b; Crama & Schyns, 2003; Orús et al., 2019).

The method of moments (MM) is a technique for estimating the parameters of a statistical model. It works by finding values of the parameters that result in a match between the sample moments and the population moments (as implied by the model). The MM is used as quick information about the unknown parameter or an initial guess for a numerical search for MLE estimates (Malá et al., 2022). Various researchers conducted the estimates of various statistical distributions based on MM. It includes the work of Hosking et al. (1985), who used the method of probability-weighted moments to derive estimators of the parameters and quantiles of the generalised extreme-value distribution. Greenstein et al. (1999) used MM to estimate the Ricean K-factor from measured power versus time, which is relatively cumbersome and time-consuming. Chang (2011) used MM compared with other numerical methods in estimating Weibull parameters for wind energy applications based on Monte Carlo simulation and analysis of actual wind speed. Bílková (2012) compare MM with L-moments based on lognormal distribution. Rocha et al. (2012) analysed and compared 7 (seven) numerical methods to assess effectiveness in determining the parameters for the Weibull distribution using wind speed data. Munkhammar et al. (2017) proposed a procedure based on the is based on MM, which is set up algorithmically to aid applicability in estimating *N*th-degree polynomial approximations to be unknown (or known) probability density functions (PDFs) based on *N* statistical moments from each distribution. Tizgui et al. (2017) compare moment methods with other estimation methods for Weibull parameters in the Agadir region in Morocco. Chaurasiya et al. (2018) examine the effectiveness of nine different numerical methods in calculating the parameters of Weibull distribution for wind power density. Honore et al. (2020) introduce measures on how each moment contributes to the precision of parameter estimates in generalised MM settings.

This study aims to demonstrate the SA algorithm's efficacy in estimating the parameters of the GGD modelled on the transformed MIRR as formulated in Sayed and Sabri (2022) compared with the MM. The data relates to long-term investments in the stocks of 62 publicly listed Malaysian property companies from 2010 to 2019, considering varying cases of one to eight-year investment periods.

MATERIALS AND METHODS

Modified Internal Rate of Return

The MIRR model in this paper was introduced by Sabri and Sarsour (2019) and developed by Abubakar and Sabri (2021a) and Sayed and Sabri (2022). The MIRR model framework is a good fit for a long-term investment. Therefore, it requires a comprehensive search to collect more in-depth financial information about the companies under study, such as the yearly updated dividend rate, stock issuance and daily updated stock price. It is crucial to focus on the form of the dividend declared for the shareholders because that tends to differ from one company to another, thus affecting the accumulation of their share unit g.

In the investment theory, some companies are found practising the share split. For example, suppose each ordinary share is split into 2 (*i. eg* = 2). In that case, the number of the share units earned by the shareholder will be multiplied by two, thereby reducing the stock price to half to enhance the liquidity of the share capital traded in the market without any change in the investor's capital. On the contrary, the bonus issue in which the company distributes the accumulated shares to the shareholders depends on the number of their shares. For instance, in a bonus issue of one share for two existing ordinary shares, the company distributes half of the accumulative share units to 1.5 share units at the end of the year. With such a bonus issue, the shareholder will earn $2 \times (1 + \frac{1}{2} \times 1) = 3$ that makes $g = 3$.

Furthermore, companies issue treasury shares or mandatory treasury shares in which the company distributes dividends for shareholders in cash, which does not affect the accumulation of the share unit g. If the company does not announce any share issuance for any year, then $g = 1$. The accumulation of share units g for a particular year is calculated by adding any share split or bonus issue in the same year and any treasury shares activity.

The long-term investment strategy associated with the MIRR model requires holding the cash out at the same level as at the beginning of every year for *k* years and reinvesting the cash dividend to increase the share units. After the desired investment's time ends, the investor earns the capital with the profit of the investment after *k* years. If the investor gets less cash than the total contribution, then the MIRR may come negative. The process of calculating the MIRR is represented in a few steps as follows:

1. Set the investment time *K*.
2. Calculate the respective terminal investment *F(K)* (Equation 1), which is the terminal value of the invested fund at the end of year *K*,

$$F(K) = S_K^{(2)} P_{u_{K+1,2}} + B_K + DIV_K \tag{1}$$

where $u_{K,1}$ is the date of share purchased and sold; $u_{K,2}$ is the date of dividend and share issued based on the stock reported for year *k*; $P_{u_{k,2}}$ is the stock price at the date $u_{K,2}$; B_K is the cash balance of year *k*; DIV_K is the cash dividend at year *k*, and *r* is the MIRR and C is the yearly fixed contribution used in Equation 2. Assuming the NPV at time zero is equal to zero, computed for MIRR as follows,

$$NPV = [S_K^{(2)} P_{u_{K+1,2}} + B_K + DIV_K](1+r)^{-\frac{u_{K+1,1}-u_{1,1}}{365}} - C\sum_{k=1}^{K+1} a_k (1+r)^{-\frac{u_{k,1}-u_{1,1}}{365}} \tag{2}$$

For the year $k = 1,\ldots,K$, $S_K^{(2)}$ is the accumulated share unit after share issuance at the end of year *k*, defined as Equation 3,

$$S_K^{(2)} = g_k \times s_k^{(1)} \tag{3}$$

where g_k is the function of share issuance, and $s_k^{(1)}$ is the share units at the beginning of year *k*.

In a long-term investment based on the MIRR model, choosing the promising stock or the right time for investing is ineffective since the MIRR differs when holding a stock for an extended investment period. Also, tracking the best investment timing is difficult, as the MIRR measure can only be spotted annually. Therefore, this study assumes that the MIRR for all stocks and the time for starting an investment are common. For computation purposes, we define the MIRR by R_{tiK}, where $i = 1,,,n$ stocks, $t = t_1,\ldots,t_T$ years of investment start, and *K* is the investment period so that $K \leq T$.

The investors either expect a positive return (profit) from an investment or, in the worst case, they can get zero return which leads to work with a non-negative MIRR. The non-negative transformed rate of return was introduced by Sabri and Sarsour (2019) as Equation 4:

$$X_{tiK} = 1 + R_{tiK} \tag{4}$$

where *K* is the investment period, $R_{tiK} > 1$ is defined as the MIRR, $i = 1,,,n$ stocks, $t = t_1,\ldots,t_T$ years of investment start, and *K* is the investment period. The non-negative transformed MIRR X_{tiK}, being a random variable, should be distributed with a non-negative distribution such as the GGD, which will be explained in the next section.

Generalised Gamma Distribution

The non-negative MIRR or the random variable X_{tiK} for the *i*-th stock at investment year *t* over the investment period of *K* years may be distributed by the three parameters of the GGD, namely *α* and *θ* with the growth rate parameter *γ*. By letting $X_{tiK} = (1+\gamma)^{K-1}X_{ti1}$, the probability density function (PDF) of the distribution is presented as Equation 5

$$f_{X_{tiK}}(x; \alpha, (1+\gamma)^{K-1}\theta) = \begin{cases} \frac{1}{\Gamma(\alpha)}\left[\frac{1}{(1+\gamma)^{K-1}\theta}\right]^{\alpha} x^{\alpha-1} e^{-\frac{x}{(1+\gamma)^{K-1}\theta}}, x > 0 \\ 0, otherwise. \end{cases} \tag{5}$$

with the mean and the variance presented in Equations 6 and 7, respectively, as follows,

$$\mu = \alpha(1+\gamma)^{K-1}\theta \tag{6}$$

$$\sigma^2 = \alpha[(1+\gamma)^{K-1}\theta]^2 \tag{7}$$

The likelihood function of Equation 5 is defined as Equation 8,

$$L(\alpha_K, (1+\gamma)^{K-1}\theta_K|x_{tiK}) = -ln[\Gamma(\alpha_K)] - \alpha_K\, ln(\,(1+\gamma)^{K-1}\theta_K) + ln(\,x_K^{\alpha_K-1}) - \frac{x_K}{(1+\gamma)^{K-1}\theta_K} \tag{8}$$

$$l(\alpha_K, (1+\gamma)^{K-1}\theta_K\big|x_{tiK}) = -\sum_{K-1}^{K^*} ln[\Gamma(\alpha_K)] - \sum_{K=1}^{K^*}[\alpha_K\, ln(\,(1+\gamma)^{K-1}\theta_K)] + \sum_{K=1}^{K^*} ln(\,x_K^{\alpha_K-1}) - \sum_{K-1}^{K^*}\frac{x_K}{(1+\gamma)^{K-1}\theta_K}. \tag{9}$$

Method of Moment (MM)

The MM is one of the existing methods used in estimating the parameters of the statistical distribution function. However, estimating the three parameters of Equation 5 using the MM is not ideal because of the difficulty of constructing the values of the three parameters from the mean and variance equations unless one of the values is fixed and solved for the other two. Therefore, in this study, we fix the value of the growth rate parameter γ to estimate the shape parameter α and the scale parameter θ using the MM. The fixed value of the growth parameter γ should be selected precisely to provide us with a closer estimation of α and θ to their true values. For the period of investment, K, the MM estimates of μs and σs can be defined as Equations 10 and 11:

$$\alpha\theta = \frac{\sum\sum_{i=1}^{n} x_i}{n} = m_{1K} \tag{10}$$

$$\alpha\theta^2 + (\alpha\theta)^2 = \frac{\sum\sum_{i=1}^{n} x_i^2}{n} = m_{2K} \tag{11}$$

Thus, it gives the following Equations 12 and 13

$$\hat{\theta} = \frac{m_{2K} - m_{1K}^2}{m_{2K}} \tag{12}$$

$$\hat{\alpha} = \frac{m_{1K}^2}{m_{2K} - m_{1K}^2} \tag{13}$$

Simulated Annealing Algorithm (SA)

The Simulated Annealing (SA) algorithm is a heuristic technique proposed separately by Kirkpatrick et al. (1983) and Cerny (1985). It is one of the most popular metaheuristics algorithms that rely on logic and rules to optimise model parameters. This process is based on Physical Annealing, which mimics the physical melting process of heating a material to its melting point and then slowly cooling it to achieve the required structure. The SA algorithm optimises by systematically decreasing the temperature and minimising the search region.

Estimating the parameters using the SA algorithm requires selecting values of control parameters, such as the temperature parameter and the initial set of the modelling parameters that meet the study purpose. Since this study focuses on estimating the GGD parameters to maximise the likelihood function, choosing a high enough temperature parameter is necessary. The likelihood function should be multiplied by (-1) to be appropriately maximised. Such persuading can be done using MATLAB programming. The SA algorithm adopted in this paper was explained in-depth in Abubakar and Sabri (2021a) and Idris and Muhammad (2022) and summed up here as follows:

(i) *Start the function folder with a big enough sample of data X and the problem function f = (Y,X) where Y is the maximum likelihood function.*
(ii) *Select controlling parameters for the SA, for example,* S_0, S_1, S_2, S_3 *such that while* $S_1 > S_0$, $S_1 = S_2 \times S_1$
(iii) *Generate random values a, b, and c within the initial lower and upper bounds.*
(iv) *Compute the likelihood function (L) at a, b, and c using Equation 8*
(v) *Generate neighbour values* a_1, b_1, and c_1 *within the initial lower and upper bounds.*
(vi) *Compute the likelihood function* L_1 *at* a_1, b_1, and c_1.
(vii) *If* $L_1 > L$ *then* $L_1 = L$ *and* $a = a_1$, $b = b_1$, and $c = c_1$.
(viii) *Else generate a random value* $m \in (0,1)$.
(ix) -$if\, e^{-\frac{(L_1-L)}{S_1}} > m$ *then* $a = a_1$, $b = b_1$, and $c = c_1$.
(x) *Print a, b, and c, which are α, θ, and γ.*

Data Collection and Experiment

The data in this study was extracted from property companies based on the Malaysian market from 2010 to 2019. The company's financial data and stock prices were collected from trusted resources, such as the Bursa Malaysia website (*www.bursama-laysia.com)* and The Wall Street Journal website *(www.wsj.com).* The data analysis involved Excel modelling, starting from listed companies' historical stock prices, the dividends declared yearly, and their share issuance for calculating the MIRR in annuity form for each year of the ten years using Equation 2. After that, the collected MIRR data was transformed into a non-negative form according to Equation 4. Since the study supports a long-term investment method, the investor is expected to hold on to a share for a minimum of one year to a maximum of ten years. Each data sample with different sizes was obtained (620, 558, 496, 434, 372, 310, 248, 186).

This study aims at modelling the MIRR data distribution on the assumption of GGD. There is a need for a good estimator to estimate the distribution parameters (α, θ, γ). In this experiment, the two parameters α and θ of the distribution are estimated using the SA and MM, considering the fixed values (-0.0001, 0.0001) of the third parameter γ to get an initial impression of the parameters. Based on this information, the SA algorithm explained above estimates the three parameters of our model. We can finally determine our SA algorithm's effectiveness by obtaining parameters close enough to the initial parameters to maximise the likelihood function and minimise the variance nicely. To measure the closeness of the estimated parameters to the initial ones, we compute the mean absolute errors (MAE) and root mean square errors (RMSE) for each investment period of the estimated parameters according to Equations 14 and 15 respectively,

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|\hat{V}_i - V\right| \quad (14)$$

$$RMSE_{V_i} = \sqrt{\frac{\sum(\hat{V}_i - V)^2}{n}}, \quad (15)$$

where $i = 1,,,n$ is the number of observations, V is the observed value and $\hat{V}$ is the predicted value of the distribution.

RESULTS AND DISCUSSION

The parameters of the GGD are estimated based on the transformed modified internal rate of return datasets using a SA in comparison with MM for investment modelling. The estimated results are presented in Tables 1 and 2. The results in Tables 1 and 2 are plotted in Figures 1 and 2 for further analysis.

Table 1 shows the initial parameters estimation of the transformed MIRR distributed using GGD as in Equation 1, with two unknown parameters α and θ and the fixed growth parameter γ = - 0.0001, 0, 0.001 over eight years investment periods of 62 property sector stocks in Malaysia from 2010 to 2019, using the method of the moment. From Table 1, it is observed that the estimated values α and θ are close to the actual values. Moreover, the mean, variance and maximum-likelihood function values are very close to different growth rate parameter values for each investment period, thus providing a steady initial assumption of the parameters generated.

Table 2 displays the estimated values for the three parameters GGD using SA. According to the trends, it is noticed that the larger the data set (Investment periods), the closer the estimated parameters are to the initial parameters. It is further observed that the estimated parameters based on SA maximise the maximum-likelihood function and minimise the variance in most cases, emphasising the efficiency of using the SA algorithm in estimating the parameters for the GGD based on MIRR data from the Malaysian property sector.

Figure 1 depicts the trends of the MAE values of the estimation methods used in this study in estimating the parameters of GGD using MIRR data from the property sector in

Table 1
Estimated parameters via method of moment

	Investment Periods							
K	1	2	3	4	5	6	7	8
n_K	620	558	496	434	372	310	248	186
m_{1K}	1.12037	1.07290	1.06335	1.05949	1.04194	1.02725	1.01692	1.01046
m_{2K}	1.67559	1.26850	1.19862	1.17247	1.12006	1.07969	1.05239	1.03564
				$\gamma = 0$				
$E(X_K)$	1.1204	1.0729	1.0633	1.0595	1.0419	1.0273	1.0169	1.0105
$Var(X_K)$	0.4204	0.1174	0.0679	0.0499	0.0344	0.0244	0.0183	0.0146
α_K	2.9861	9.8058	16.6495	22.4730	31.5448	43.1858	56.5871	69.8467
θ_K	0.3752	0.1094	0.0639	0.0471	0.0330	0.0238	0.0180	0.0145
$l(\alpha_K, \theta_K, \gamma_K)$	-416.81	-115.37	3.8792	57.630	108.29	136.876	142.16	125.92
				$\gamma = -0.0001$				
$E(X_K)$	1.1204	1.0727	1.0629	1.0589	1.0411	1.0262	1.0157	1.0090
$Var(X_K)$	0.4204	0.1173	0.0679	0.0499	0.0344	0.0244	0.0182	0.0146
α_K	2.9861	9.8058	16.6495	22.4730	31.5448	43.1858	56.5871	69.8467
θ_K	0.3752	0.1094	0.0639	0.0471	0.0330	0.0238	0.0180	0.0145
$l(\alpha_K, \theta_K, \gamma_K)$	-416.81	-115.37	3.8785	57.629	108.28	136.870	142.15	125.90
				$\gamma = 0.0001$				
$E(X_K)$	1.1204	1.0731	1.0638	1.0638	1.0428	1.0283	1.0181	1.0119
$Var(X_K)$	0.4204	0.1174	0.0680	0.0680	0.0345	0.0245	0.0183	0.0147
α_K	2.9861	9.8058	16.649	22.473	31.544	43.1858	56.587	69.846
θ_K	0.3752	0.1094	0.0639	0.0472	0.0330	0.0238	0.0180	0.0145
$l(\alpha_K, \theta_K, \gamma_K)$	-416.81	-115.37	3.8785	57.629	108.28	136.870	142.15	125.90

Note. $n_K = n\ (T - K + 1)$; $n = 62$ and $T = 8$. Furthermore, $l(\alpha_K, \theta_K, \gamma_K)$ indicate log-likelihood

Table 2
Estimated parameters via simulated annealing

	Investment Periods							
K	1	2	3	4	5	6	7	8
n_K	620	558	496	434	372	310	248	186
α_K	2.9299	9.6941	17.3998	22.5085	32.4969	43.6111	57.2080	70.2149
θ_K	0.3709	0.1071	0.0610	0.0473	0.0321	0.0235	0.0177	0.0143
γ_K	-0.000098	-0.000085	-0.000098	-0.00009	-0.00009	-0.00009	-0.00009	-0.00009
$E(X_K)$ 1.0866	1.0384	1.0619	1.0652	1.0432	1.0244	1.0128	1.0045	
$Var(X_K)$ 0.4030	0.1112	0.0648	0.0504	0.0335	0.0241	0.0179	0.0144	
$l(\alpha_K, \theta_K, \gamma_K)$-421.107	-119.065	5.0878	57.5151	108.3712	136.807	142.003	125.672	

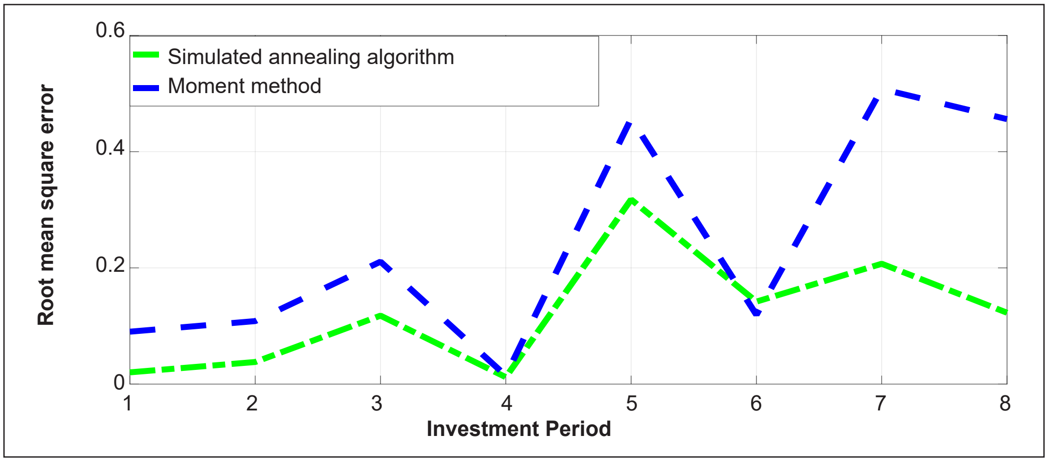

Figure 1. Comparing MAE of the estimation methods

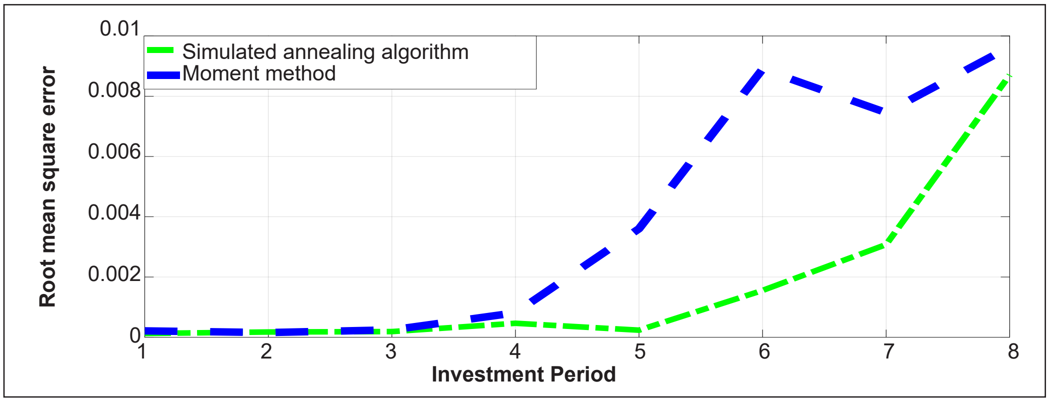

Figure 2. Comparing the $RMSE_{Vi}$ of the estimation methods

Malaysia. The estimated parameters in both SA and MM approaches are close to the initial parameters as the values of their MAE approaches are zero. The SA approach's performance improved with the investment period increase.

In Figure 2, the performance of SA and MM is displayed in terms of their RMSE accumulations throughout the investment periods. According to the trends, it can be observed that both the SA and MM have lower RMSE accumulation which is close to zero. The two methods' understudy exhibits similar trends as the initial investment period but disagrees as the investment period grows. The SA algorithm utilised to estimate the GGD parameters provides efficient results by displaying lower error accumulation even with a larger sample size. Moreover, the longer the investment period, the lower the RMSE because holding on to a stock for a longer period minimises the variance and hence the risk recommended in stock investment. The SA employs an optimisation strategy similar to how metallurgy and glass are produced. SA varies from most other iterative improvement algorithms in that, depending on a pseudo-temperature variable, candidate solutions of poorer quality than present can be allowed during the algorithm's iterations. However, SA's performance is better than MM's, especially when the sample size is large (long-term investment period). It indicates that SA can be used for parameter estimation when a large sample is used. The process is relatively easy and fast compared to traditional estimation methods.

CONCLUSION

This study's results demonstrate the SA's capability to estimate the parameters of the GGD using the transformed MIRR data. It reveals how the SA parameters are close to the initial ones and can be utilised to maximise the likelihood function and minimise the variance. Moreover, comparing the values of the SA estimated parameters with the initial parameters validates our long-term investment strategy. Such an algorithm performs better with the initial assumptions about the parameters. We used the MM to estimate our model's shape and scale parameters while keeping the growth rate parameter values fixed. It provided us with suitable initial parameters to use with SA. The results reveal the efficacy of the SA algorithm in parameter estimation problems. Hence, the algorithm can be applied to multiple generalised distributions belonging to the same family, thereby assisting in resolving modelling difficulties associated with real-world data. In addition, the performance of SA can be improved by hybridising with other estimation methods, such as the Election algorithm, variable neighbourhood search, and differential evolution algorithm, to achieve more robust and more efficient implementations.

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